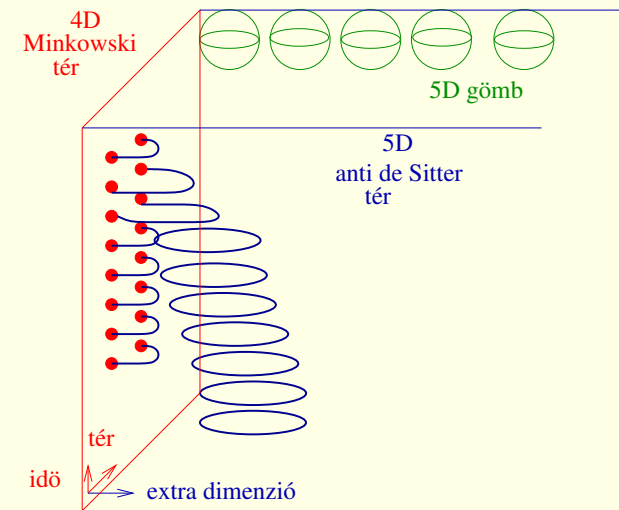
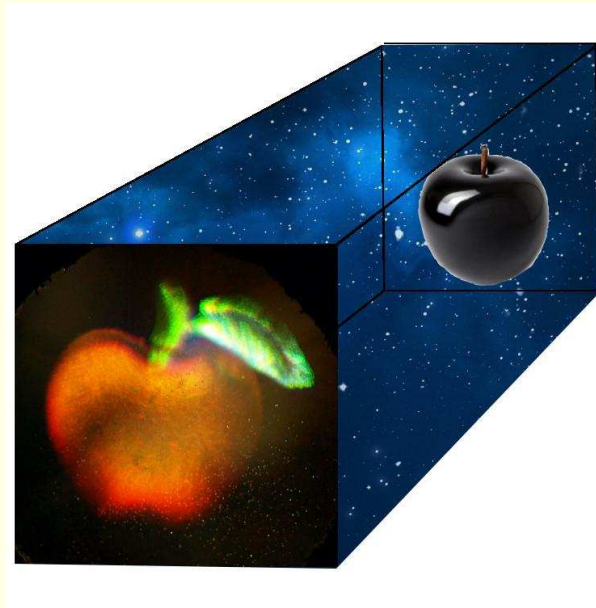
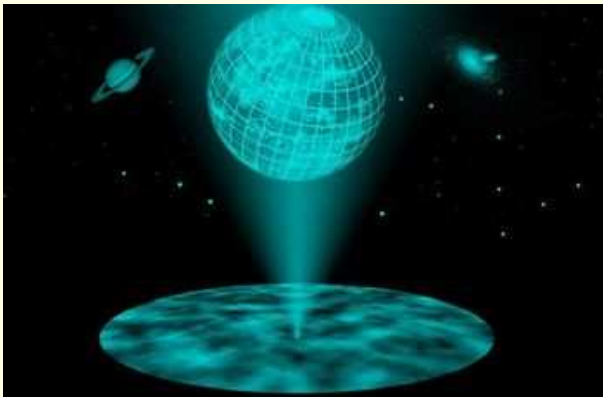


Atomoktól a csillagokig: 2017. október 12.

# Holográfia a részecskefizikában

Bajnok Zoltán

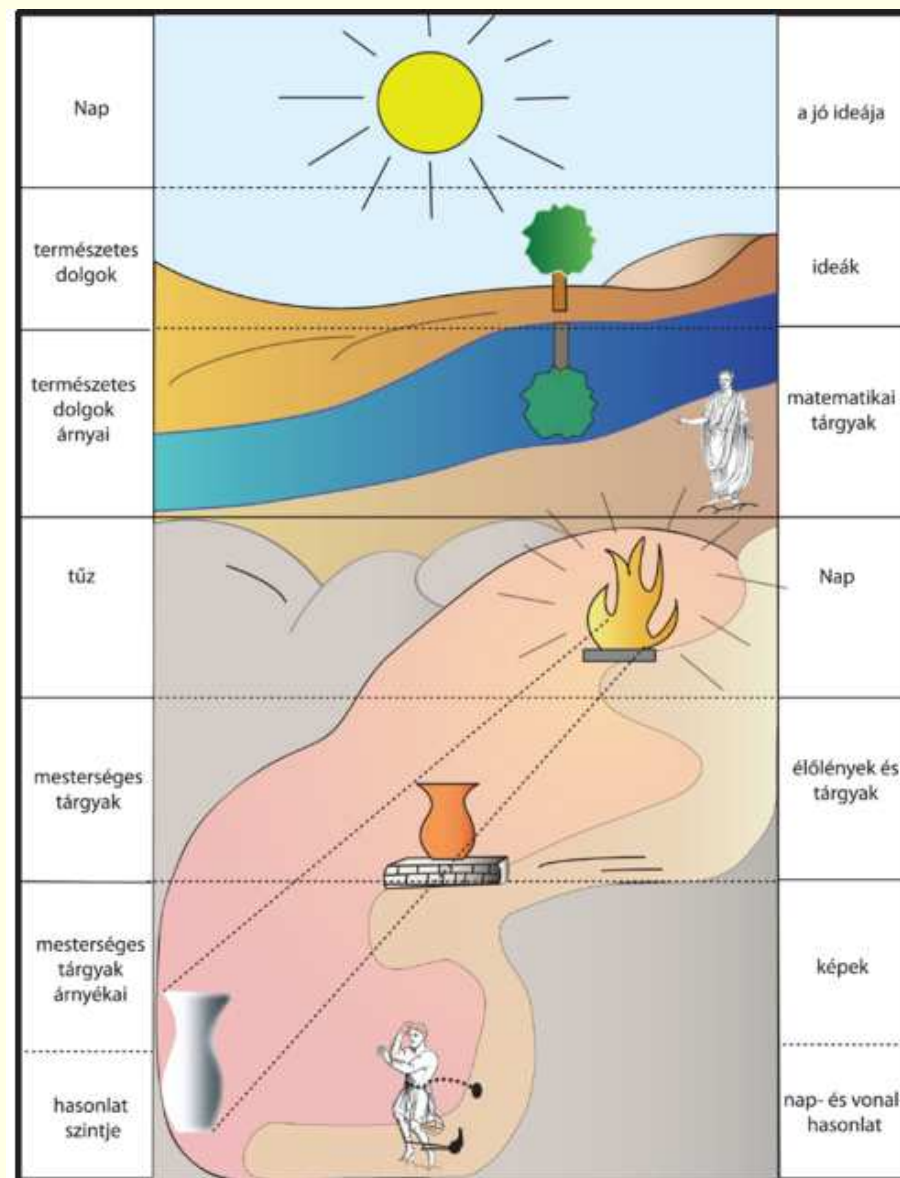
MTA, Wigner Fizikai Kutatóközpont



Hány dimenziós a világunk? Hogyan lehetne a dimenziót megmérni?

# Barlang hasonlat

Már a görögök is gondolkodtak a problémán: Platón

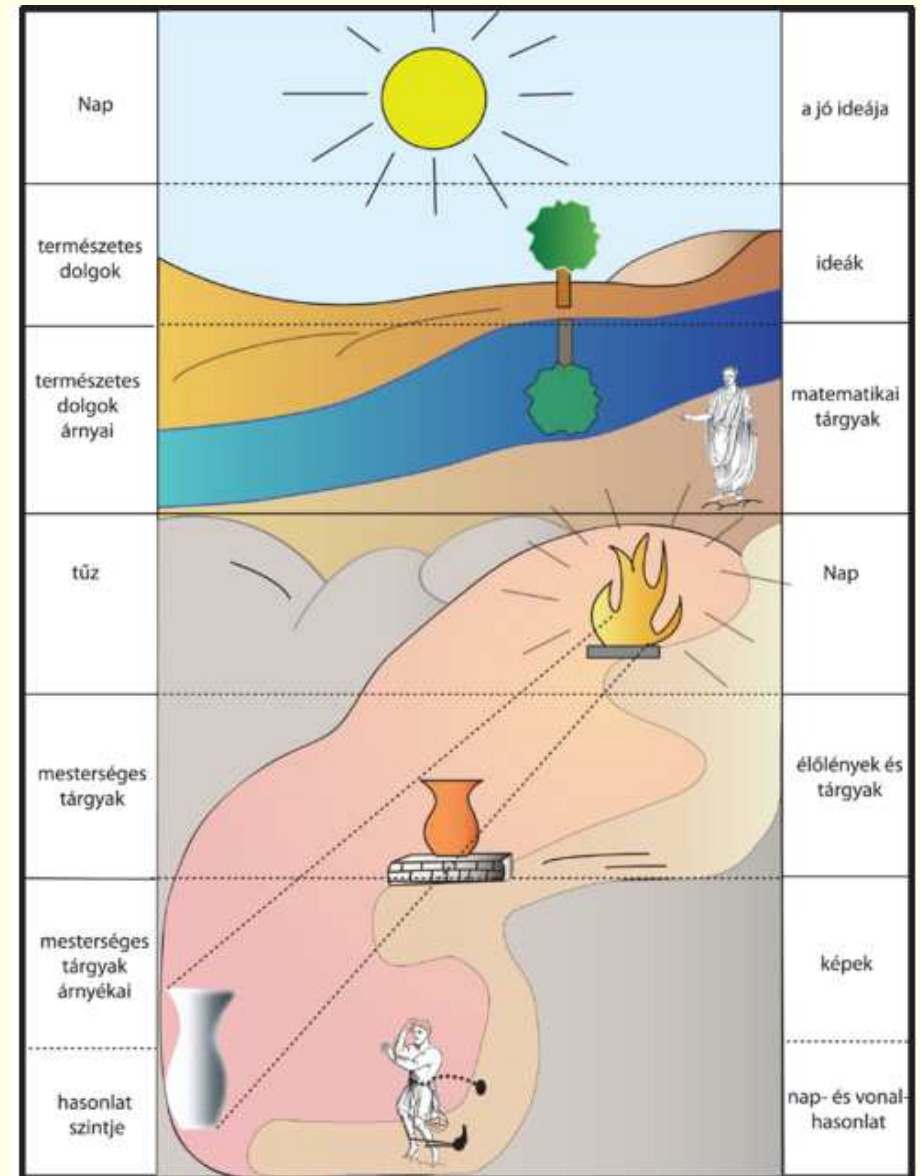


# Barlanghasonlat

Már a görögök is gondolkodtak a problémán: Platón

Mi van ha a háromdimenziós világunk sem az igazi, van egy magasabb dimenziós világ, aminek, mi csak az árnyékai vagyunk.

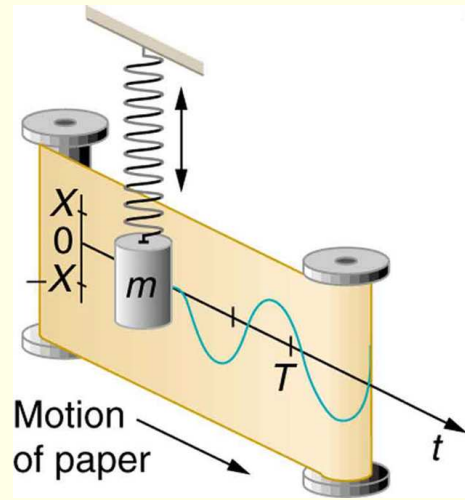
Hogyan tudnánk fizikai kísérletekkel kimutatni az extra dimenziót?



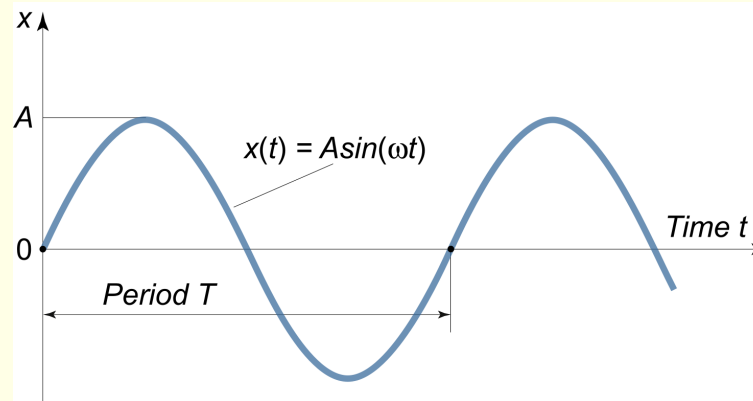
**Figyelmeztetés!**

# Figyelmeztetés!

## Harmonikus rezgés

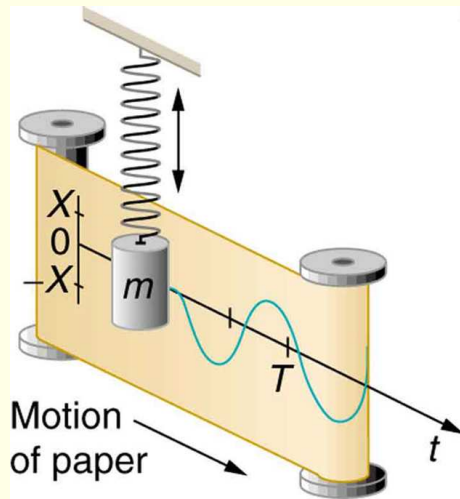


$$x(t) = A \sin(\omega t)$$

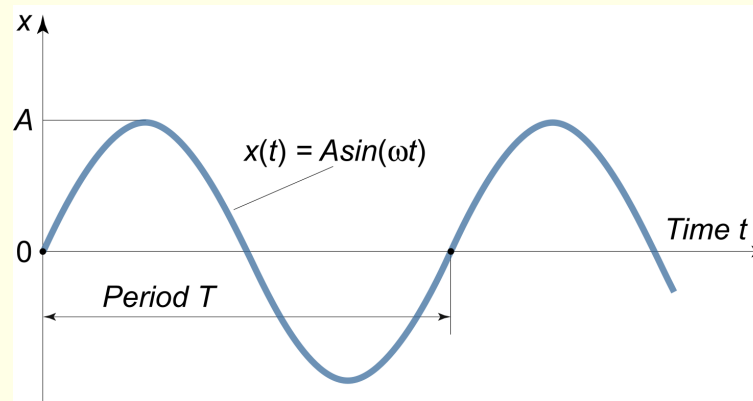


# Figyelmeztetés!

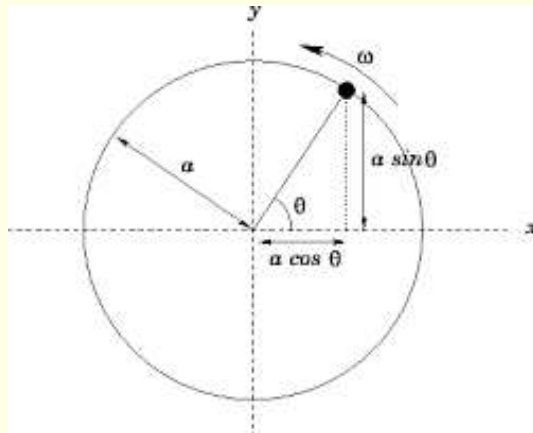
## Harmonikus rezgés



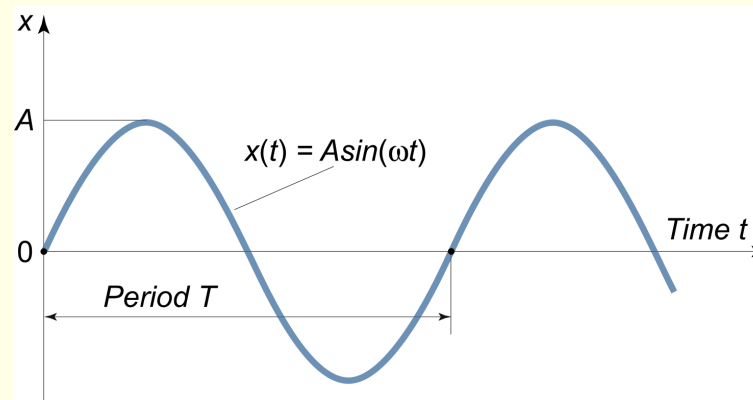
$$x(t) = A \sin(\omega t)$$



## Egyenletes körmozgás és vetülete



$$\theta = \omega t$$



Egyenletes körmozgás a barlang falán harmonikus rezgőmozgásnak látszik!  
Ezek ekvivalens, megkülönböztethetetlen leírások. ( Habár az egyik egyszerűbb,  
mint a másik.)

# Bolygómozgás

Planetárium vagy háromdimenziós mozgás?



Mars mozgása





# Bolygómozgás

Planetárium vagy háromdimenziós mozgás?



Mars mozgása



Newton: mozgásegyenlet

$$\begin{aligned} F_x &= ma_x = m \frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x} \\ F_y &= ma_y = m \frac{d^2y}{dt^2} = -\frac{\partial V}{\partial y} \\ F_z &= ma_z = m \frac{d^2z}{dt^2} = -\frac{\partial V}{\partial z} \end{aligned}$$



gravitációs potenciál

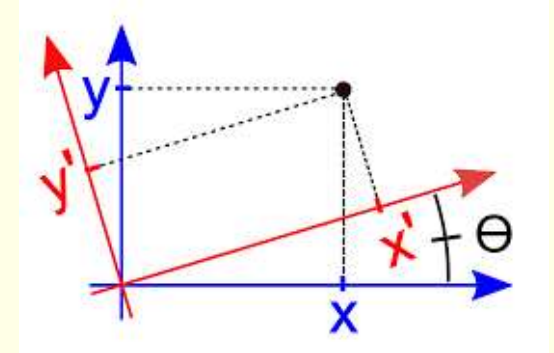
$$V(r) = -\gamma \frac{mM}{r} = -\gamma \frac{mM}{\sqrt{x^2 + y^2 + z^2}}$$



# Dimenzió az egyenletek alakjából

Forgassuk el a koordináta rendszerünket  $z' = z$  körül

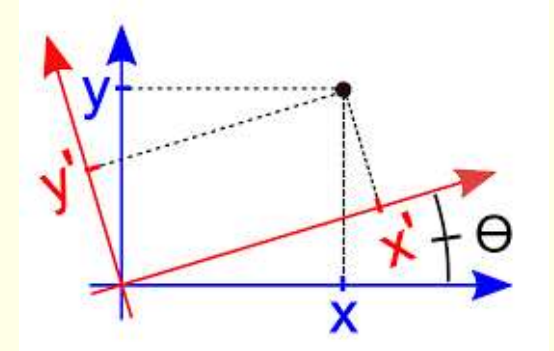
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta x + \sin \theta y \\ -\sin \theta x + \cos \theta y \end{pmatrix} \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



# Dimenzió az egyenletek alakjából

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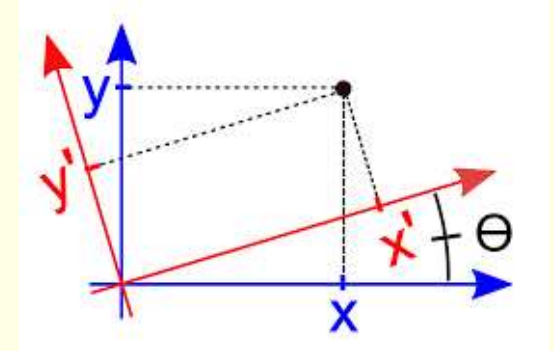
A mozgásegyenlet alakja nem változik

$$\begin{pmatrix} m \frac{d^2 x}{dt^2} = -\frac{\partial V}{\partial x} \\ m \frac{d^2 y}{dt^2} = -\frac{\partial V}{\partial y} \\ m \frac{d^2 z}{dt^2} = -\frac{\partial V}{\partial z} \end{pmatrix} \rightarrow \begin{pmatrix} m \frac{d^2 x'}{dt^2} = -\frac{\partial V}{\partial x'} \\ m \frac{d^2 y'}{dt^2} = -\frac{\partial V}{\partial y'} \\ m \frac{d^2 z'}{dt^2} = -\frac{\partial V}{\partial z'} \end{pmatrix}$$

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A kovariancia transzformációk a háromdimenziós tér hossztartó lineáris leképezései:  $SO(3)$  (komplex tér  $SU(3)$ )

$$O_z = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, O_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{pmatrix}, O_y = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix}$$

A Newton egyenlet 3d jelölésben:  $m \frac{dx_i^2}{dt^2} = F_i = -\frac{\partial V}{\partial x_i} \quad i = 1, 2, 3$   
 $(x_1, x_2, x_3) = (x, y, z)$

# Egyenletek egyszerűsége

Forgó koordináta rendszerben

$$\begin{aligned} \mathbf{a} &= \frac{d^2 \mathbf{r}}{dt^2} = \frac{d}{dt} \frac{d\mathbf{r}}{dt} = \frac{d}{dt} \left( \left[ \frac{d\mathbf{r}}{dt} \right] + \boldsymbol{\omega} \times \mathbf{r} \right) \\ &= \left[ \frac{d^2 \mathbf{r}}{dt^2} \right] + \boldsymbol{\omega} \times \left[ \frac{d\mathbf{r}}{dt} \right] + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt} \\ &= \left[ \frac{d^2 \mathbf{r}}{dt^2} \right] + \boldsymbol{\omega} \times \left[ \frac{d\mathbf{r}}{dt} \right] + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times \left( \left[ \frac{d\mathbf{r}}{dt} \right] + \boldsymbol{\omega} \times \mathbf{r} \right) \\ &= \left[ \frac{d^2 \mathbf{r}}{dt^2} \right] + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + 2\boldsymbol{\omega} \times \left[ \frac{d\mathbf{r}}{dt} \right] + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}). \end{aligned}$$

Polár koordinátákban

$$\begin{aligned} \mathbf{r} &= \mathbf{r}(t) = r \hat{\mathbf{e}}_r \\ \mathbf{v} &= v \hat{\mathbf{e}}_r + r \frac{d\theta}{dt} \hat{\mathbf{e}}_\theta + r \frac{d\varphi}{dt} \sin \theta \hat{\mathbf{e}}_\varphi \\ \mathbf{a} &= \left( a - r \left( \frac{d\theta}{dt} \right)^2 - r \left( \frac{d\varphi}{dt} \right)^2 \sin^2 \theta \right) \hat{\mathbf{e}}_r \\ &\quad + \left( r \frac{d^2 \theta}{dt^2} + 2v \frac{d\theta}{dt} - r \left( \frac{d\varphi}{dt} \right)^2 \sin \theta \cos \theta \right) \hat{\mathbf{e}}_\theta \\ &\quad + \left( r \frac{d^2 \varphi}{dt^2} \sin \theta + 2v \frac{d\varphi}{dt} \sin \theta + 2r \frac{d\theta}{dt} \frac{d\varphi}{dt} \cos \theta \right) \hat{\mathbf{e}}_\varphi \end{aligned}$$

# Az elemek periódusos rendszere

Periodic Table of the Elements © www.elementsdatabase.com

1 H																	2 He																												
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne																												
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar																												
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr																												
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe																												
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn																												
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn																																				
<table border="1" style="width: 100%; text-align: center;"> <tr> <td>58 Ce</td><td>59 Pr</td><td>60 Nd</td><td>61 Pm</td><td>62 Sm</td><td>63 Eu</td><td>64 Gd</td><td>65 Tb</td><td>66 Dy</td><td>67 Ho</td><td>68 Er</td><td>69 Tm</td><td>70 Yb</td><td>71 Lu</td> </tr> <tr> <td>90 Th</td><td>91 Pa</td><td>92 U</td><td>93 Np</td><td>94 Pu</td><td>95 Am</td><td>96 Cm</td><td>97 Bk</td><td>98 Cf</td><td>99 Es</td><td>100 Fm</td><td>101 Md</td><td>102 No</td><td>103 Lr</td> </tr> </table>																		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu																																
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr																																

Minek az árnyéka a periódusos rendszer?

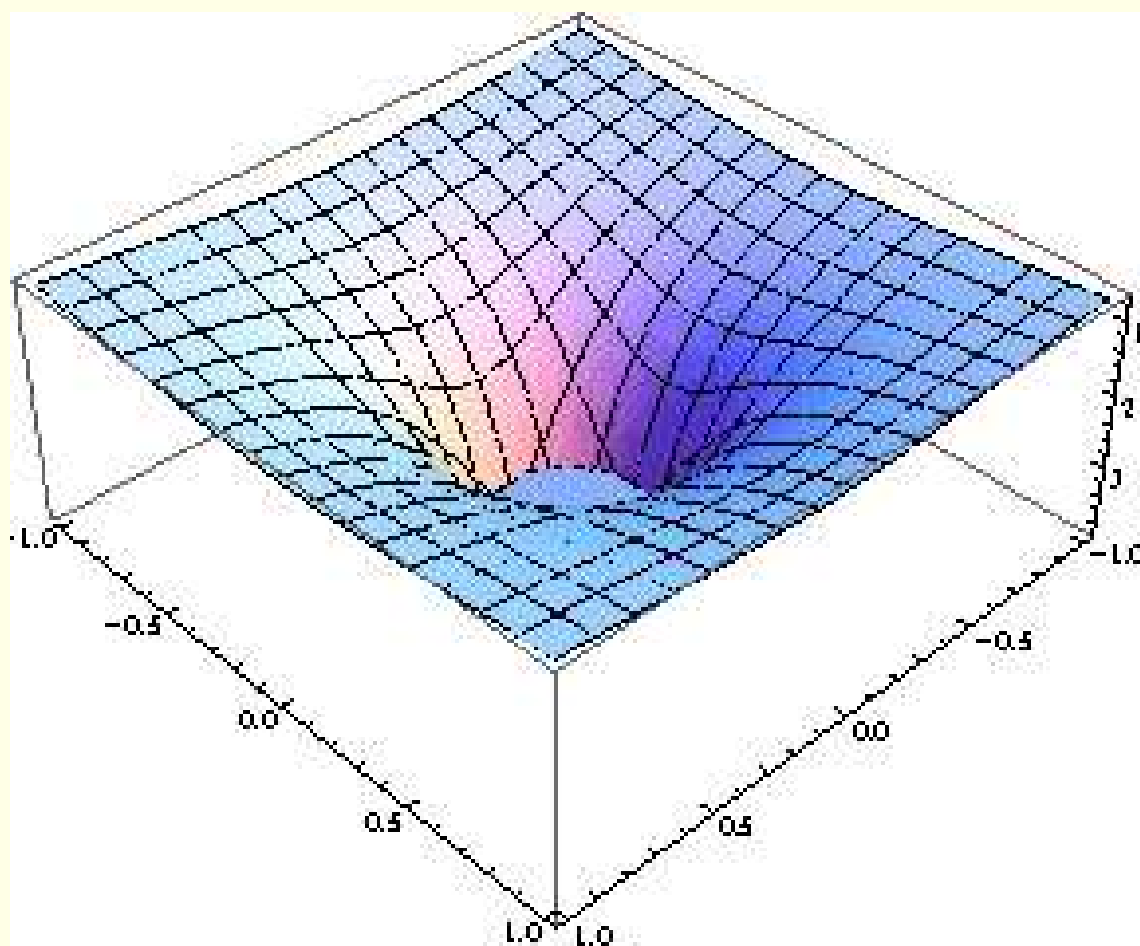
# Dimenzió leolvasása a spektrumból

Elektromos kölcsönhatás (potenciális energia  $V(r) = k\frac{Zq}{r}$ )



## Dimenzió leolvasása a spektrumból

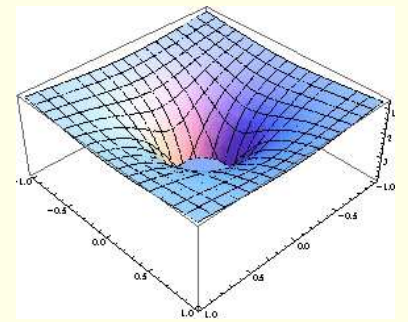
Elektromos kölcsönhatás (potenciális energia  $V(r) = k\frac{Zq}{r}$ )



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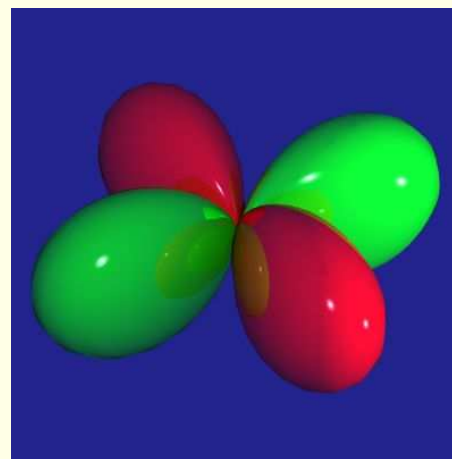
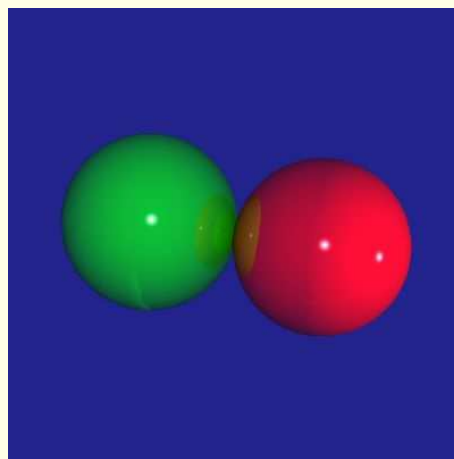
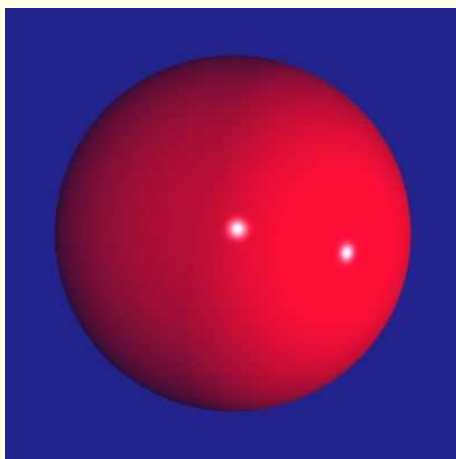
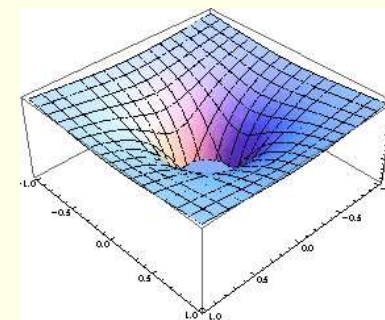
Kvantum mechanika



# Dimenzió leolvasása a spektrumból

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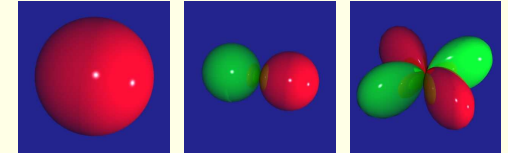
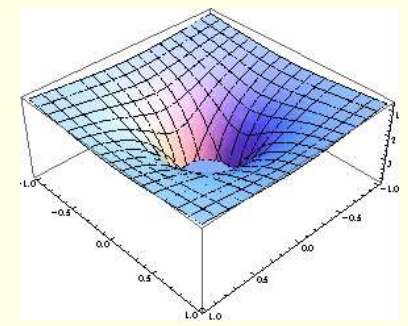
Kvantum mechanika



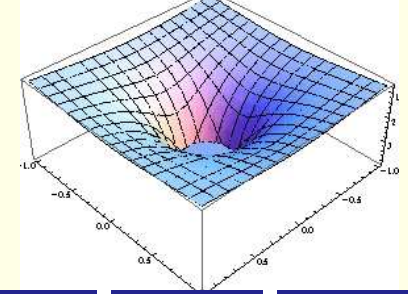
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Kvantum mechanika

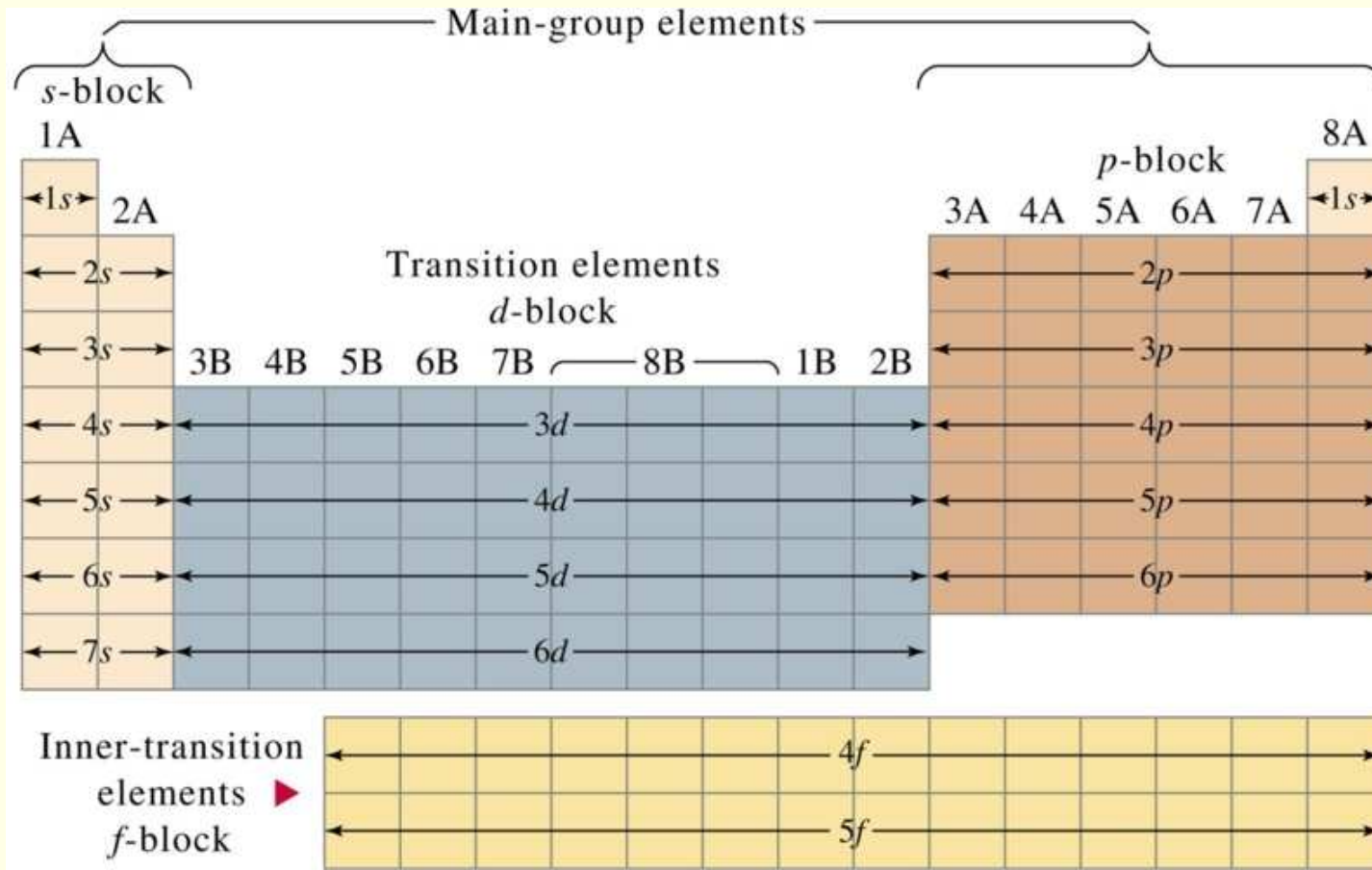
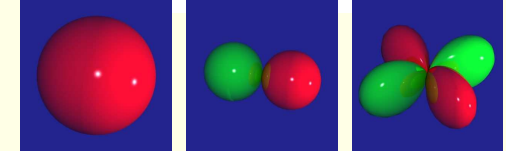


# Dimenzió leolvasása a spektrumból

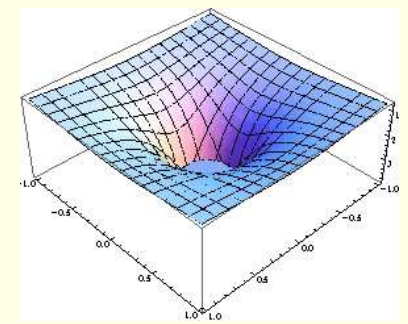


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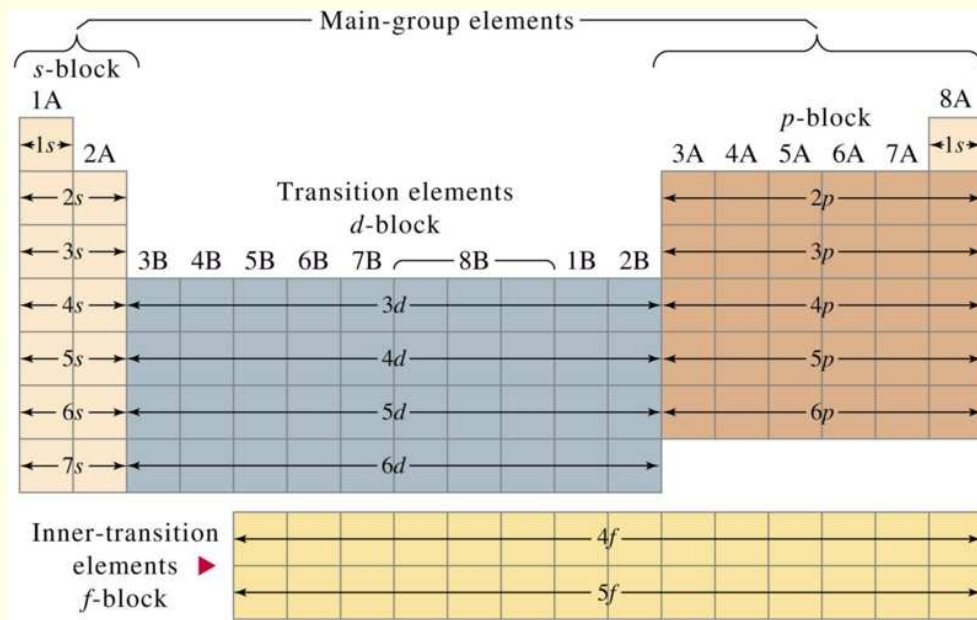
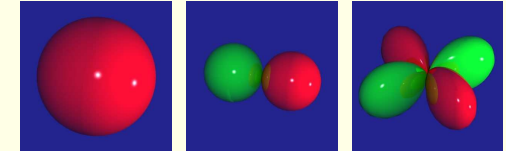


# Dimenzió leolvasása a spektrumból



Elektromos kölcsönhatás (potenciális energia  $V(r) = k \frac{Zq}{r}$ )

Kvantum mechanika



Periodic Table of the Elements

© www.elementsdatabase.com

- hydrogen
- alkali metals
- alkali earth metals
- transition metals
- poor metals
- nonmetals
- noble gases
- rare earth metals

1																	2
H																	He
3	4											5	6	7	8	9	10
Li	Be											B	C	N	O	F	Ne
11	12											13	14	15	16	17	18
Na	Mg											Al	Si	P	S	Cl	Ar
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
87	88	89	104	105	106	107	108	109	110								
Fr	Ra	Ac	Unq	Unp	Unh	Uns	Uno	Une	Unn								
		58	59	60	61	62	63	64	65	66	67	68	69	70	71		
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
		90	91	92	93	94	95	96	97	98	99	100	101	102	103		
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Dimenzió: p-elemek = 2 × dimenzió



# Elektromos és mágneses kölcsönhatás egyesítése



$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

# Elektromos és mágneses kölcsönhatás egyesítése



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Maxwell egyenletek:

$$\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \quad \nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \cdot \mathbf{E} = \sum_i \frac{\partial E_i}{\partial x_i} = \partial_i E_i$$

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} = 0$$

# Elektromos és mágneses kölcsönhatás egyesítése



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

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Kovariancia transzformációk: forgatások + Lorentz transzformációk

$$x' = \frac{(x+vt)}{\sqrt{1-\frac{v^2}{c^2}}}; t' = \frac{(t+\frac{xv}{c^2})}{\sqrt{1-\frac{v^2}{c^2}}} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

# Elektromos és mágneses kölcsönhatás egyesítése



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Maxwell egyenletek:

$$\begin{aligned}\nabla &= \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) & \nabla \cdot \mathbf{E} &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\ & & \nabla \cdot \mathbf{E} &= \sum_i \frac{\partial E_i}{\partial x_i} = \partial_i E_i \\ & & \frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} &= 0\end{aligned}$$

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$$x' = \frac{(x+vt)}{\sqrt{1-\frac{v^2}{c^2}}}; t' = \frac{(t+\frac{xv}{c^2})}{\sqrt{1-\frac{v^2}{c^2}}} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Az idő a negyedik dimenzió  $(ct, x, y, z) = x^\mu \quad \mu = 1, 2, 3, 4$

$$\begin{pmatrix} \gamma & \frac{v\gamma}{c} & 0 & 0 \\ \frac{v\gamma}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \begin{pmatrix} \gamma & 0 & \frac{v\gamma}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{v\gamma}{c} & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} \gamma & 0 & 0 & \frac{v\gamma}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{v\gamma}{c} & 0 & 0 & \gamma \end{pmatrix} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

# Elektromos és mágneses kölcsönhatás egyesítése



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Maxwell egyenletek:

$$\begin{aligned}\nabla &= \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) & \nabla \cdot \mathbf{E} &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\ & & \nabla \cdot \mathbf{E} &= \sum_i \frac{\partial E_i}{\partial x_i} = \partial_i E_i \\ & & \frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} &= 0\end{aligned}$$

Kovariancia transzformációk: forgatások + Lorentz transzformációk

$$x' = \frac{(x+vt)}{\sqrt{1-\frac{v^2}{c^2}}}; t' = \frac{(t+\frac{xv}{c^2})}{\sqrt{1-\frac{v^2}{c^2}}} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

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Maxwell egyenletek ha  $(c\rho, J_x, J_y, J_z) = J^\mu$  akkor  $\partial_\mu F^{\mu\nu} = J^\nu$

$\partial^\mu F_{\mu\nu} (E \rightarrow B, B \rightarrow -E) = 0 \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  potenciál  $(\phi, A_x, A_y, A_z)$

# Melyik a fizika legszebb egyenlete?

Physics World, közvéleménykutatás:



# Melyik a fizika legszebb egyenlete?

Physics World, közvéleménykutatás:



Leonhard Euler (1707-1783)

$$e^{i\pi} + 1 = 0$$

$i, \pi, e, 1, 0$  and  $+, \cdot, \hat{}$

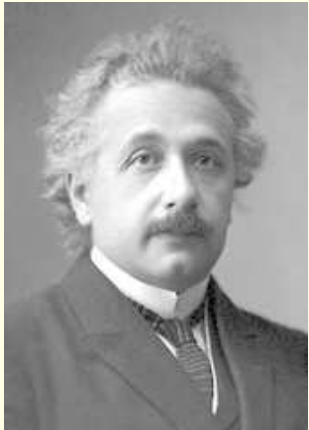


James Clerk Maxwell (1831-1879)

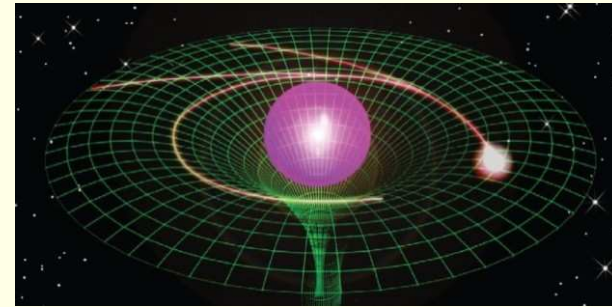
$$d \star F = j \quad ; \quad dF = 0$$

egyesítés: elektromágnesség

# Gravitáció + relativitás elmélet

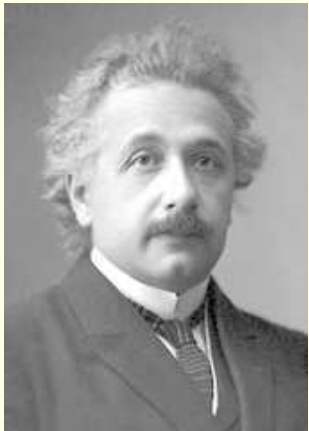


$$\begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ \cdot & g_{xx} & g_{xy} & g_{xz} \\ \cdot & \cdot & g_{yy} & g_{yz} \\ \cdot & \cdot & \cdot & g_{zz} \end{pmatrix}$$

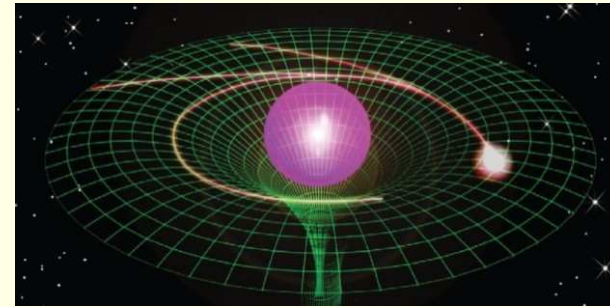


metrikus tenzor  $v \cdot u = g_{\mu\nu} u^\mu v^\nu$

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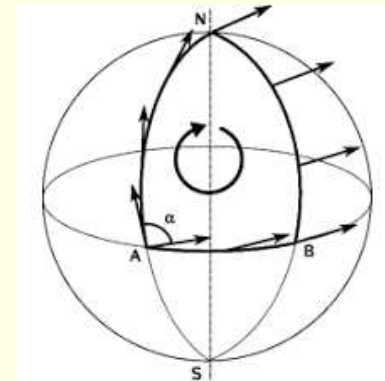


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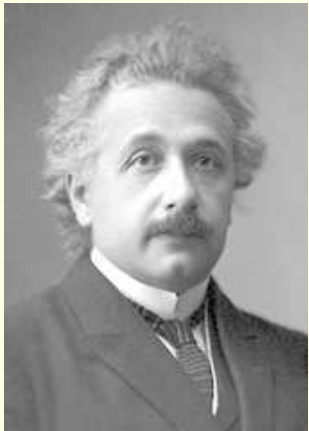


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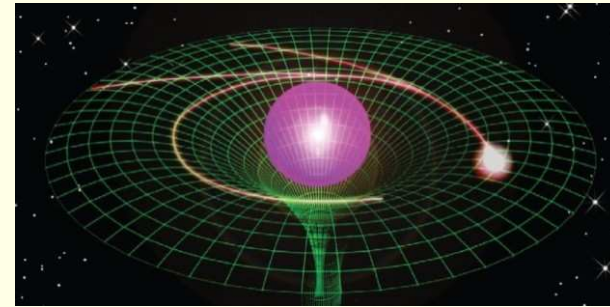
metrika  $\rightarrow$  térgömbület a párhuzamos eltolásokból



# Gravitáció + relativitás elmélet

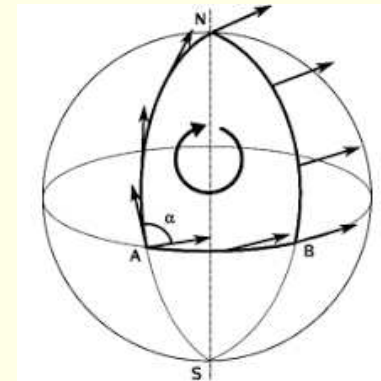


$$\begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ \cdot & g_{xx} & g_{xy} & g_{xz} \\ \cdot & \cdot & g_{yy} & g_{yz} \\ \cdot & \cdot & \cdot & g_{zz} \end{pmatrix}$$



metrikus tenzor  $v \cdot u = g_{\mu\nu} u^\mu v^\nu$

metrika  $\rightarrow$  térgörbület a párhuzamos eltolásokból



Einstein egyenlet  
görbület=anyag

Einstein's Equation

$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{curvature of spacetime}} = \underbrace{(8\pi G)}_{\text{constants}} \underbrace{T_{\mu\nu}}_{\text{energy and momentum}}$$

# Gravitációs és elektromágneses kölcsönhatás egyesítése

Mindkettőben  $V(r) = \frac{\alpha}{r}$  potenciál

DE: gravitáció = általános relativitás elmélet

Kaluza: 4+1 dimenziós gravitáció

$g_{5\mu} = (\phi, A_x, A_y, A_z)$  görbülete  $F_{\mu\nu}$

$$\begin{pmatrix} g_{55} & g_{5t} & g_{5x} & g_{5y} & g_{5z} \\ \cdot & g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ \cdot & \cdot & g_{xx} & g_{xy} & g_{xz} \\ \cdot & \cdot & \cdot & g_{yy} & g_{yz} \\ \cdot & \cdot & \cdot & \cdot & g_{zz} \end{pmatrix}$$

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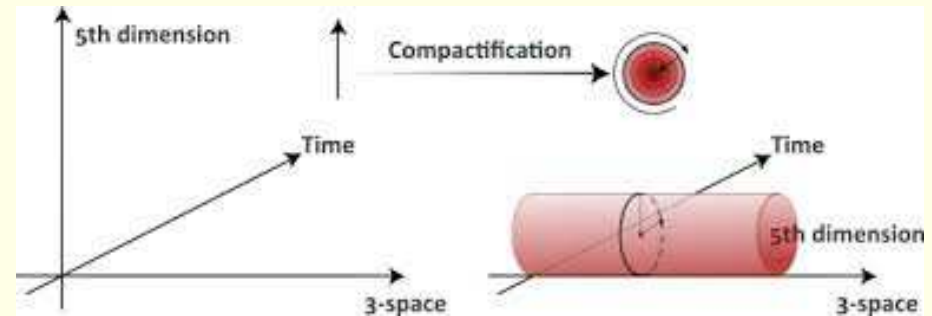
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4+1 D Einstein egyenletek = 3+1 D Einstein + Maxwell egyenletek

De miért nem látjuk?

Klein: Mert fel van csavarodva.



# Gravitációs és elektromágneses kölcsönhatás egyesítése

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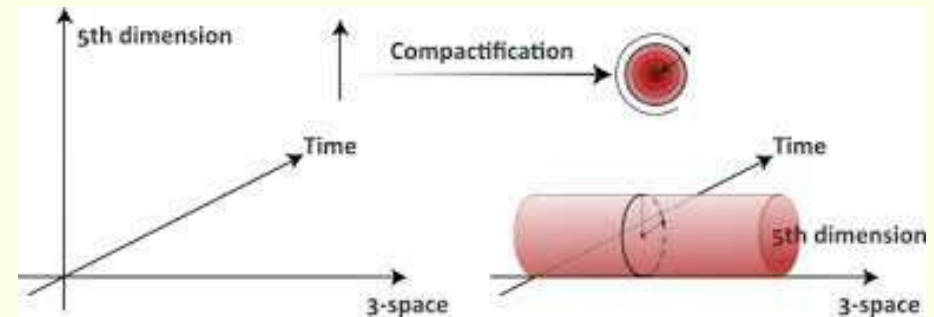
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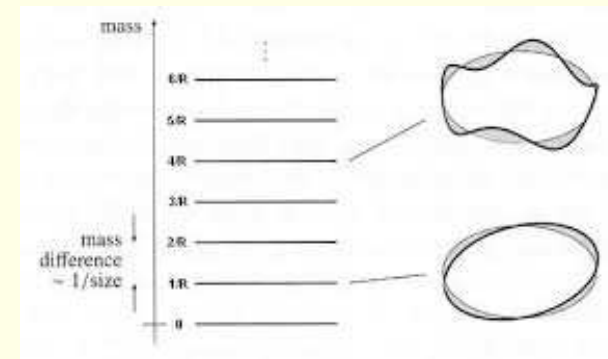


Kvantumelmélet

Impulzus kvantálás  $p = \frac{n}{R}$ ,

energiaspektrum  $E_n = \frac{n^2}{R^2}$  egyenközű

De mi van a természetben?



Miből áll a világ és hogyan hat kölcsön?



# Miből áll a világ és hogyan hat kölcsön?



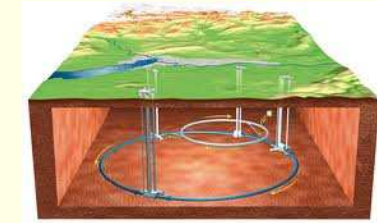
mikroszkóp



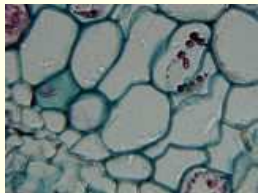
elektronmikroszkóp



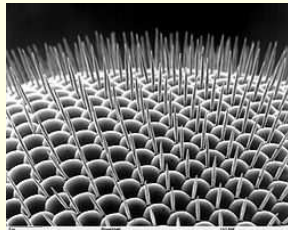
szinkrotron



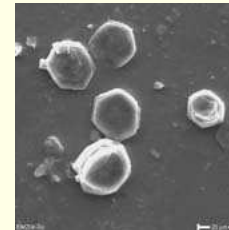
LHC



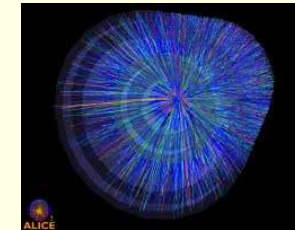
mikrométer



nanométer



sok atom



kvarkok, leptonok

# Miből áll a világ és hogyan hat kölcsön?



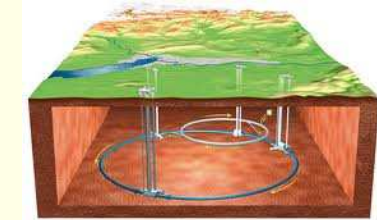
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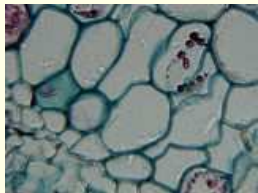
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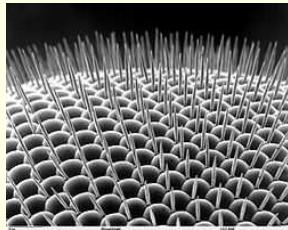
szinkrotron



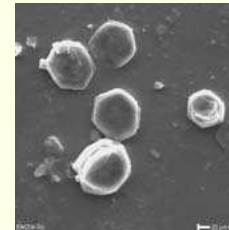
LHC



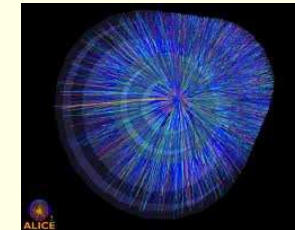
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nanométer



sok atom

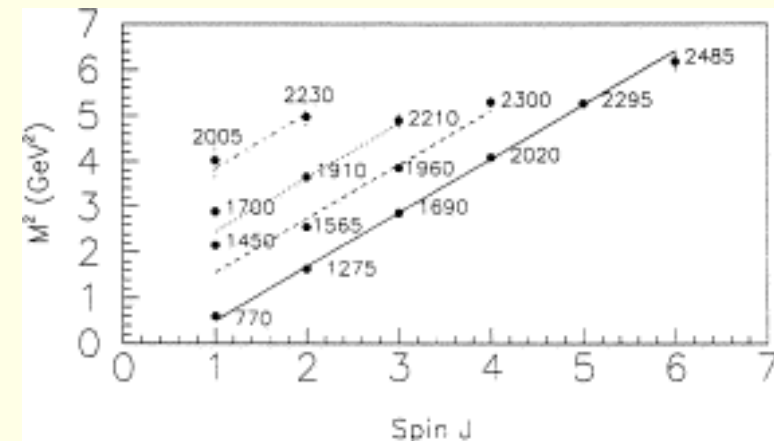


kvarkok, leptonok

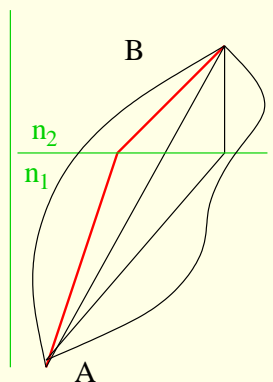
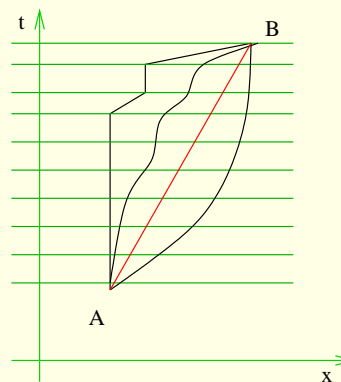
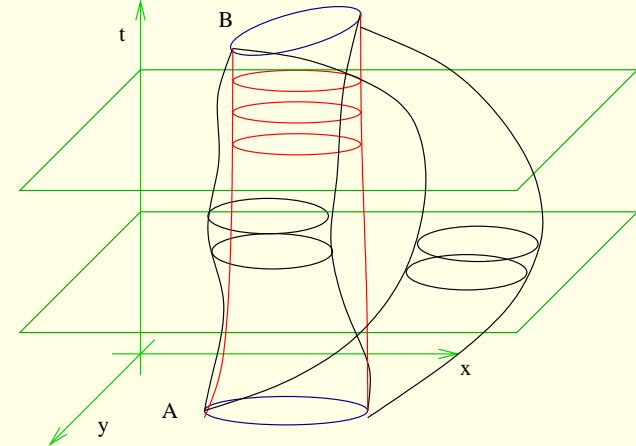
hadron spektrum

Jó közelítéssel  $M^2(J) = \alpha'J + \beta n$

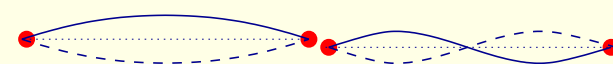
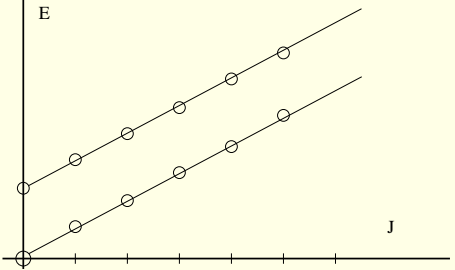
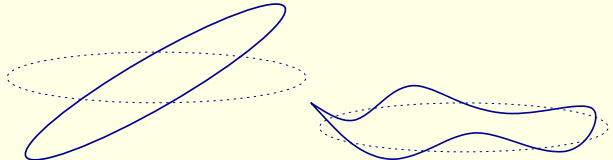
DE NEM  $M(n) = \frac{n}{R}$



# Húrok klasszikus dinamikája

fény	pontrészecske	húr
Fermat elv	téridő $(x, t)$	téridő $(x, y, t)$
		
idő minimális	téridő út minimális	téridő felület minimális

# Kvantum húrok spektruma

nyílt húr		zárt húr
		
foton, elektron, kvarkok+...	dimenzió = 10	graviton+...
mértékelmélet anyaggal	$M^2(J) = \alpha' J + \text{const.}$	gravitáció

Hogyan magyarázható a hadronspektrum?

# Hogyan magyarázható a hadronspektrum?



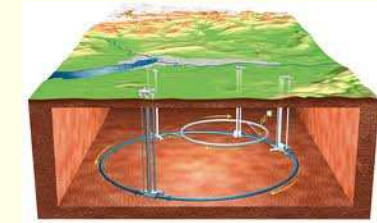
mikroszkóp



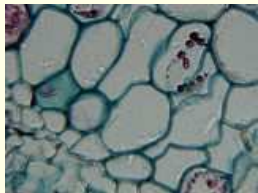
elektronmikroszkóp



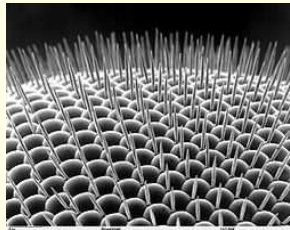
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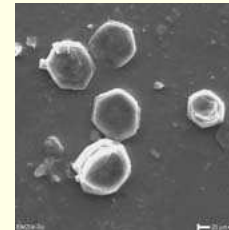
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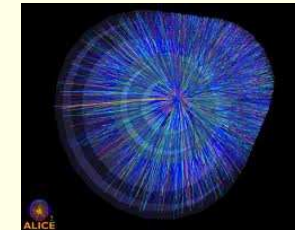
mikrométer



nanométer



sok atom



kvarkok, leptonok



# Hogyan magyarázható a hadronspektrum?



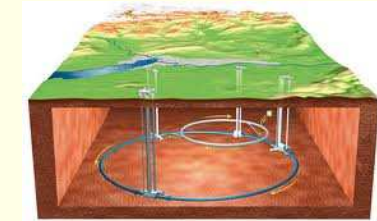
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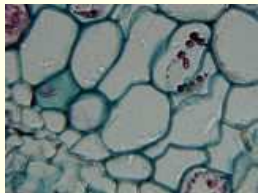
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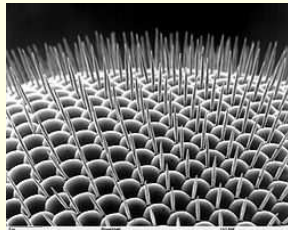
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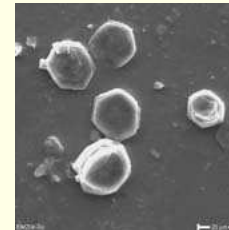
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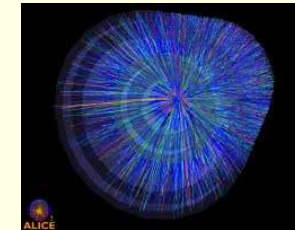
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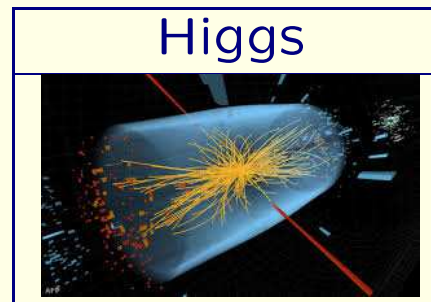
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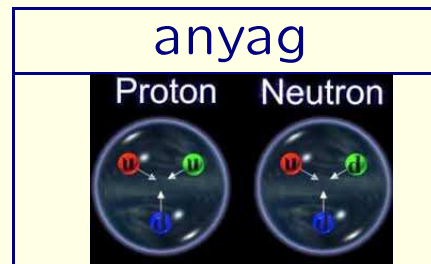
kvarkok, leptonok

periódusos rendszer

Leptons		Quarks		Bosons (Forces)	
name	mass charge spin	name	mass charge spin	name	mass charge spin
electron $e$	0.511 MeV -1 1/2	up $u$	2.4 MeV 2/3 1/2	photon $\gamma$	0 0 1
electron neutrino $\nu_e$	<2.2 eV 0 1/2	down $d$	4.8 MeV -1/3 1/2	weak force $W^\pm$	80.4 GeV 0 1
muon $\mu$	105.7 MeV -1 1/2	strange $s$	104 MeV -1/3 1/2	weak force $Z$	91.2 GeV 0 0
muon neutrino $\nu_\mu$	<0.17 MeV 0 1/2	charm $c$	1.27 GeV 2/3 1/2	strong force $g$	0 0 1
tau $\tau$	1.777 GeV -1 1/2	bottom $b$	4.2 GeV -1/3 1/2	gluon $g$	0 0 1
tau neutrino $\nu_\tau$	<15.5 MeV 0 1/2	top $t$	171.2 GeV 2/3 1/2	photon $\gamma$	0 0 1



Higgs



anyag

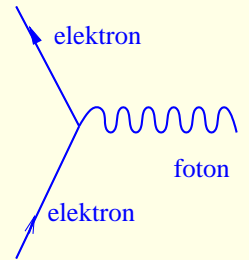
	Kölcsönhatás
$\gamma$	elektromágneses
$W^\pm, Z$	gyenge
$g$	erős
$gr$	gravitációs

# Kvantumelektrodinamika

Elektromos + mágneses kh:  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

+Kvantumelmélet → kvantumelektrodinamika

$U(1)$  mértékelmélet:  $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$



$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - e\bar{\Psi}A\Psi$$

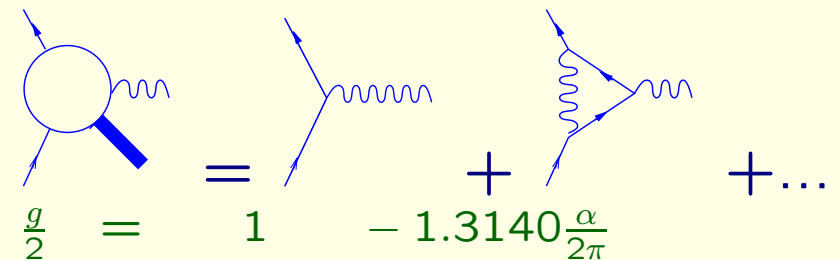
kísérleti eredmény:  $\underline{\mu} = g \frac{e\hbar}{2mc} \underline{s}$  ahol  $g = 2(1 + a)$

[Gabrielse 2006]:  $a = 1159652180.85(.76) \times 10^{-12}$

perturbáció számítás:

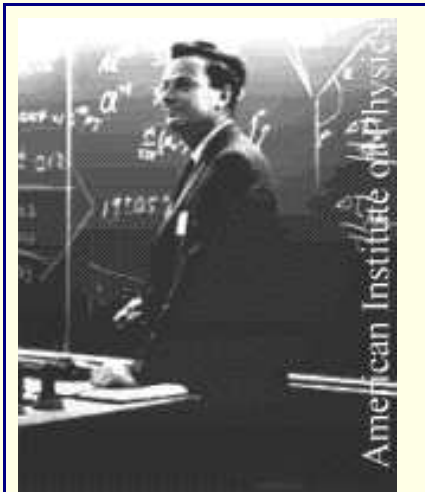
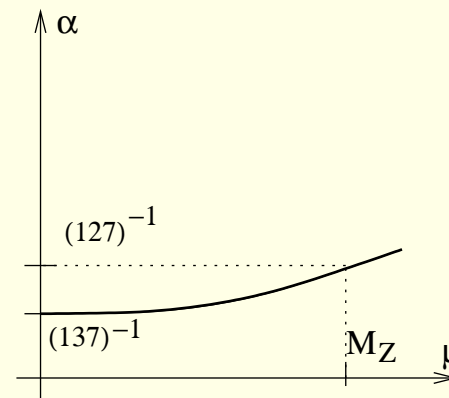
Feynman gráfok

$$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$$



impulzusfüggő csatolás:

$$\beta(\alpha) = \mu \frac{\partial \alpha}{\partial \mu} > 0$$



Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

Kvantumos

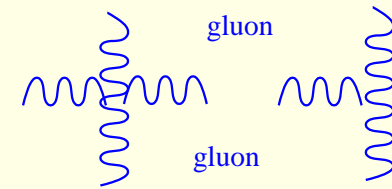
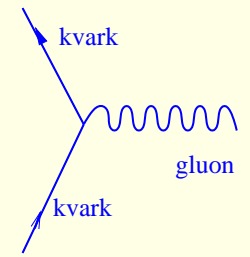
mértékelmélet

# Kvantumszíndinamika

foton  $A_\mu \leftrightarrow G_\mu^{1..8}$  gluon  $\rightarrow F_{\mu\nu}^{1..8}$

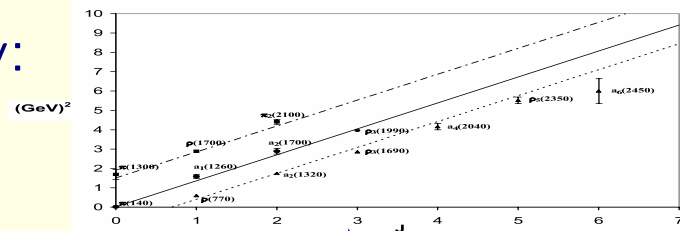
elektron  $\Psi_e \leftrightarrow \Psi_{kvark}$  kvark

$SU(3)$  mértékelmélet:  $G_\mu \rightarrow g^{-1}G_\mu g + g^{-1}\partial_\mu g$



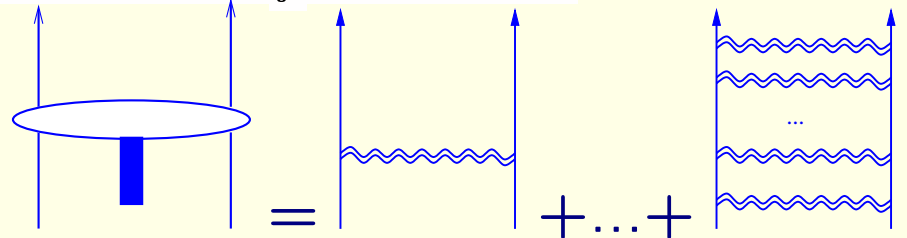
$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - g\bar{\Psi}G\Psi$$

kísérleti eredmény:  
hadron spektrum



perturbáció számítás:  
Feynman gráfok

$$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$$

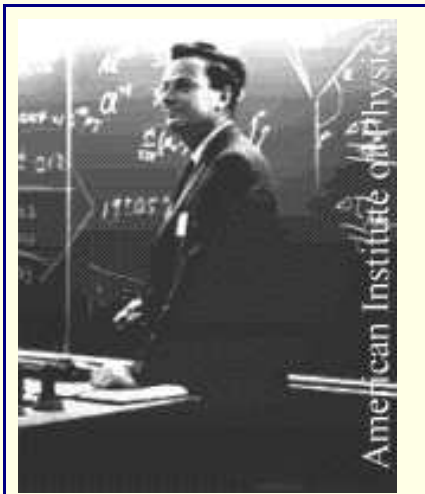
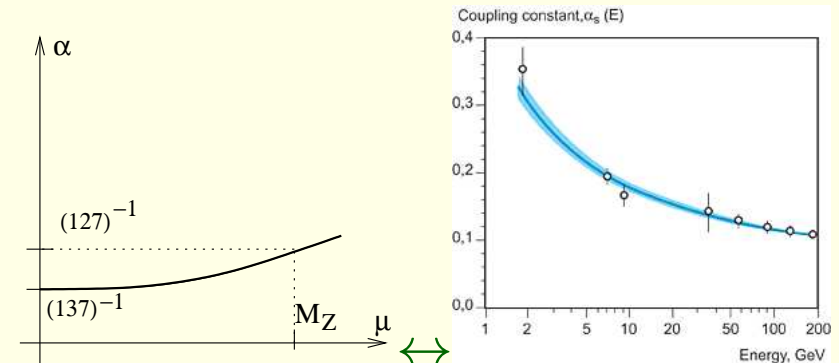


impulzusfüggő csatolás:

$$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$$

aszimptotikus szabadság

bezárás



Kvantum-  
mértékelmélet  
aszimptotikus  
szabadság

2004 Nobel Prize in Physics



David J. Gross      Frank Wilczek  
H. David Politzer



# Erős kölcsönhatás=kvantumszíndinamika



SU(3) mértékelmélet

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\cancel{D} - m)\Psi - g\bar{\Psi}G\Psi$$

Wigner Jenő:

The simplicities of natural laws arise through the complexities of the language we use for their expression.

Alacsony energia: nemperturbatív fizika

bezárás

hadronspektrum,  
nehézion ütközés

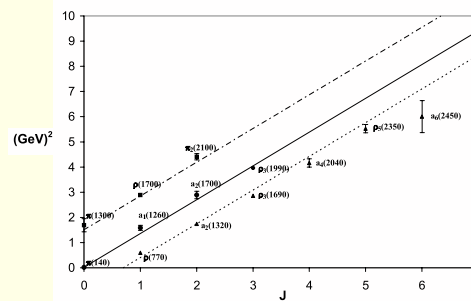
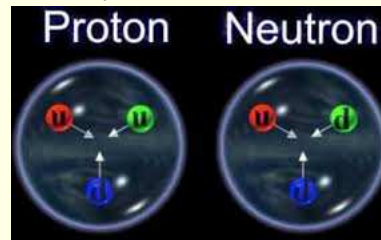
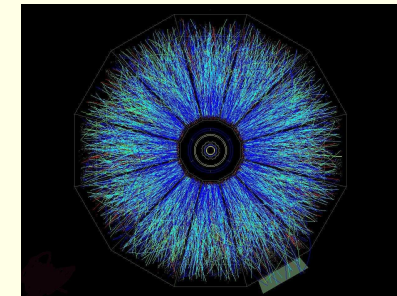
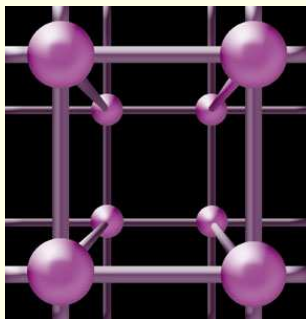


Fig. 1.



Rács-mértékelmélet:

világ  $32^4$

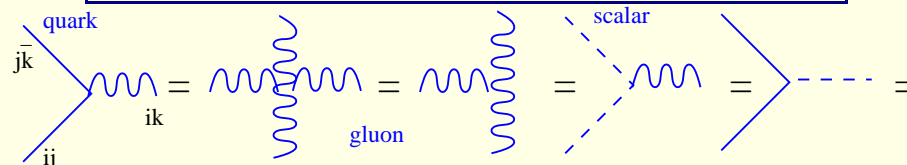


protontömeg ✓  
nehézionütközés?

Wigner: It is nice to know that the computer understands the problem. But I would like to understand it too.

egyszerűsített modell

$$\mathcal{N} = 4 \text{ D}=4 \text{ SU}(N) \text{ SYM}$$



$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\bar{\Psi}\cancel{D}\Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

# A részecskefizika standard modellje

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge}/\psi} . \quad (1)$$

Here,

$$\mathcal{L}_{\text{Dirac}} = i\bar{e}_L^i \not{\partial} e_L^i + i\bar{\nu}_L^i \not{\partial} \nu_L^i + i\bar{e}_R^i \not{\partial} e_R^i + i\bar{u}_L^i \not{\partial} u_L^i + i\bar{d}_L^i \not{\partial} d_L^i + i\bar{u}_R^i \not{\partial} u_R^i + i\bar{d}_R^i \not{\partial} d_R^i ; \quad (2)$$

$$\mathcal{L}_{\text{mass}} = -v \left( \lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_W^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ; \quad (3)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} , \quad (4)$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{WZA} &= ig_2 \cos \theta_W \left[ (W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_{\mu\nu}^+ W^{-\mu} Z^\nu - W_{\mu\nu}^- W^{+\mu} Z^\nu \right] \\ &+ ie \left[ (W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_{\mu\nu}^+ W^{-\mu} A^\nu - W_{\mu\nu}^- W^{+\mu} A^\nu \right] \\ &+ g_2^2 \cos^2 \theta_W \left( W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu \right) \\ &+ g_2^2 \left( W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu} A_\nu A^\nu \right) \\ &+ g_2 e \cos \theta_W \left[ W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W^{-\mu} Z_\nu A^\nu \right] \\ &+ \frac{1}{2} g_2^2 (W_\mu^+ W_\nu^-) (W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu}) ; \end{aligned} \quad (6)$$

and

$$\mathcal{L}_{\text{gauge}/\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 (W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu) - e A_\mu J_A^\mu , \quad (7)$$

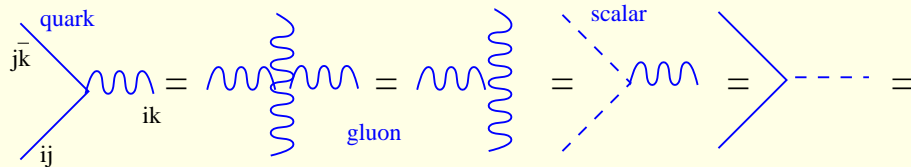
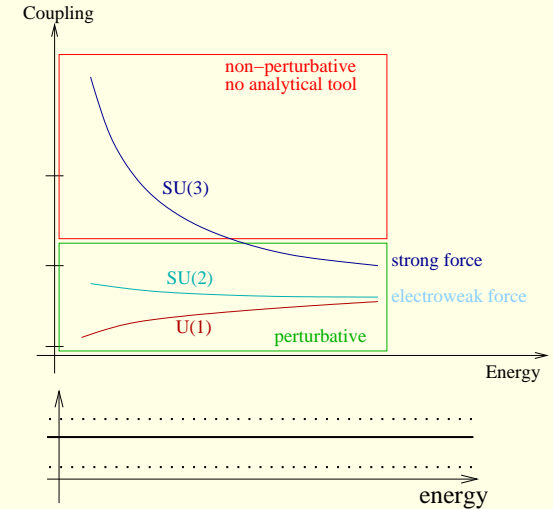
where

$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} \left( \bar{\nu}_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j \right) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[ \frac{1}{2} \bar{\nu}_L^i \gamma^\mu \nu_L^i + \left( -\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &\quad + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left( -\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \\ &\quad \left. + \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left( \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left( \frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left( -\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned} \quad (8)$$

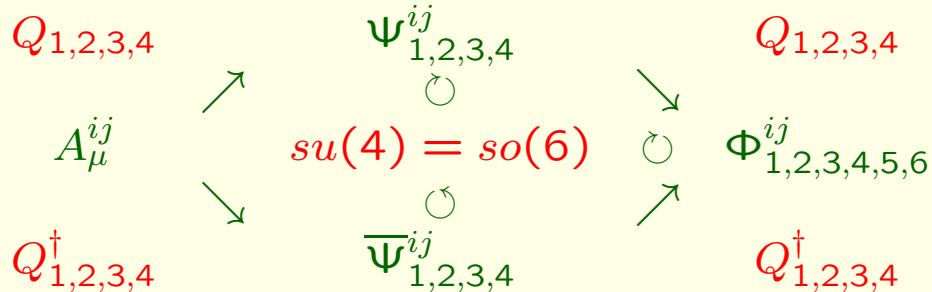
# Maximálisan (szuper)szimmetrikus mértékelmélet

## Alapvető kölcsönhatások

kölcsönhatás	részecskék	mértékcsoporthat
elektromgáneses	foton+elektron	$U(1)$
elektrogyenge	$W^\pm, Z$ $\mu, \nu$ +Higgs	$SU(2) \times U(1)$
erős	gluon+kvark	$SU(3)$
MaxSzim. SYM	gluon $A_\mu$ +kvark $\Psi$ +skalar $\Phi$	$SU(N)$



minden tér  $N^2 - 1$  komponensű mátrix

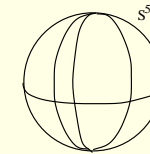


$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

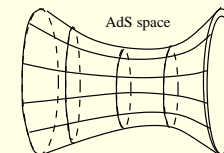
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

## Szimmetriák:

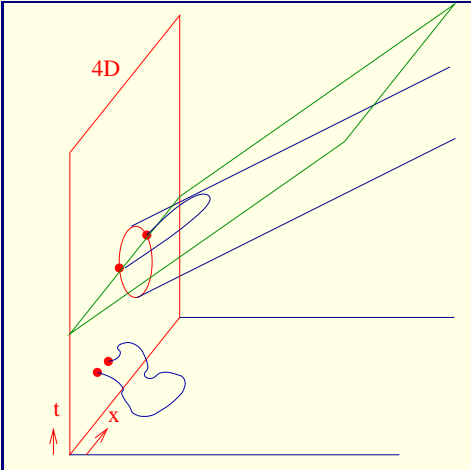
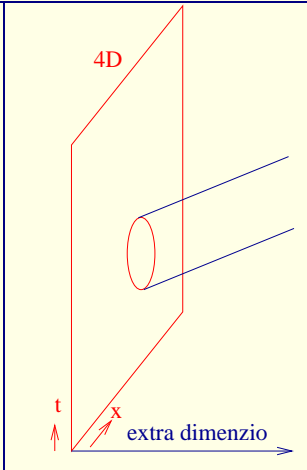
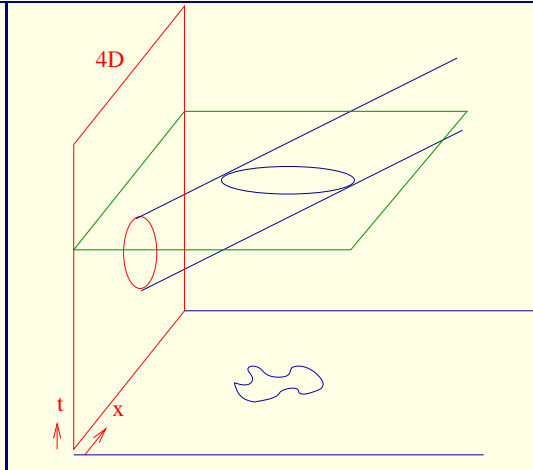
belső:  $su(4) = so(6)$   
5 dimenziós gömb



téridő: konform  $\supset$  Lorentz  
5 dimenziós Anti-de Sitter tér



# Maldacena: Mérték/gravitáció dualitás

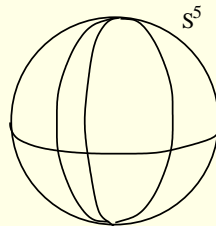
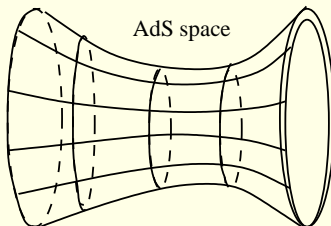
nyílt húr	idő relatív	zárt húr
		
nyílt húr folyamat		zárt húr folyamat
mértékelmélet	=	gravitáció

$\mathcal{N} = 4$  D=4  $SU(N)$  SYM

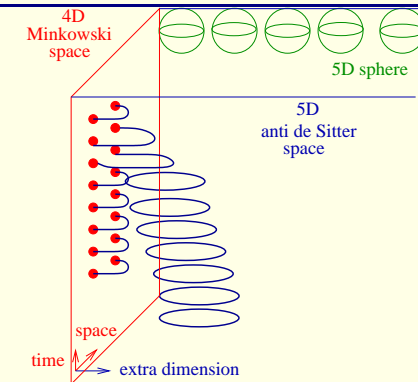
$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

szimmetriák



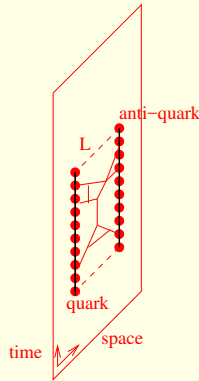
szuperhúr  $AdS_5 \times S^5$  háttéren



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

# AdS/CFT: kvark-anti-kvark potenciál

gluon kicserélődések

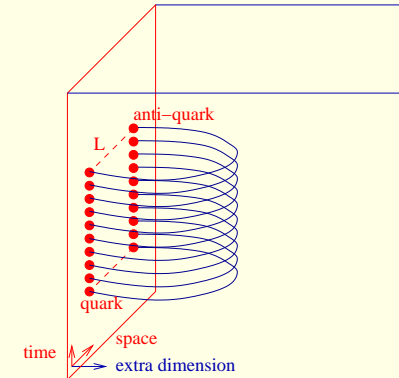


$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$

Feynman gráfokkal 4 hurok

≡

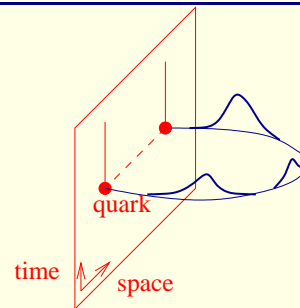
Minimális AdS felület



$$V(L) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimális AdS felület + fluktuációk

Egzakt leírás minden csatolásra



gyenge csatolás = mértékelmélet  
erős csatolás = húrelmélet

# Összegzés

Le vagyunk láncolva a 3 (+1) dimenziós Minkowski világhoz

Az extra dimenziókra csak az egyenleteink szimmetriáján keresztül következtethetünk. Lehet, hogy van több ekvivalens leírás, de mi a legegyszerűbbet választjuk.

