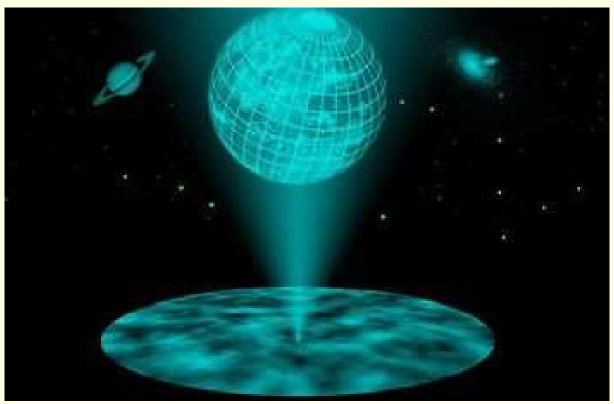
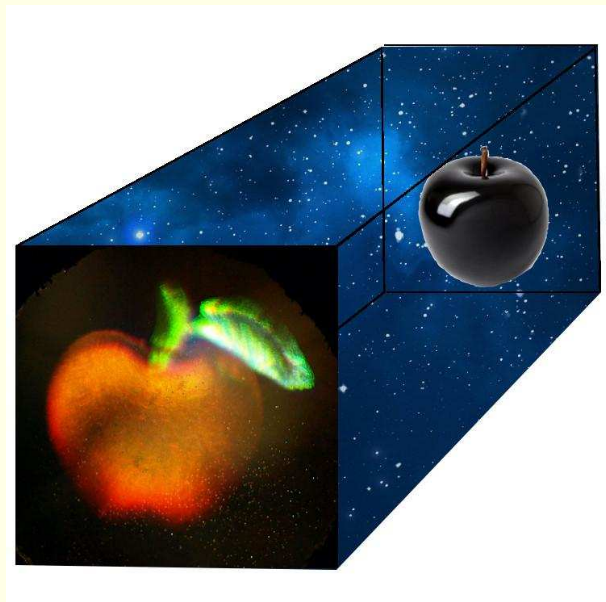
Atomoktól a csillagokig: 2017. október 12.

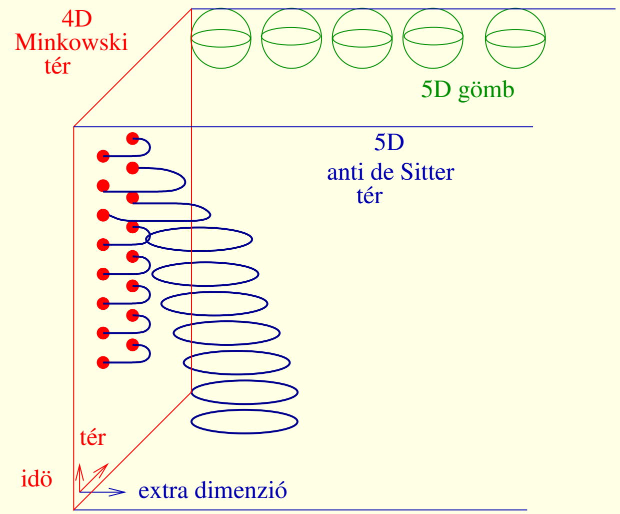
Holográfia a részecskefizikában

Bajnok Zoltán

MTA, Wigner Fizikai Kutatóközpont



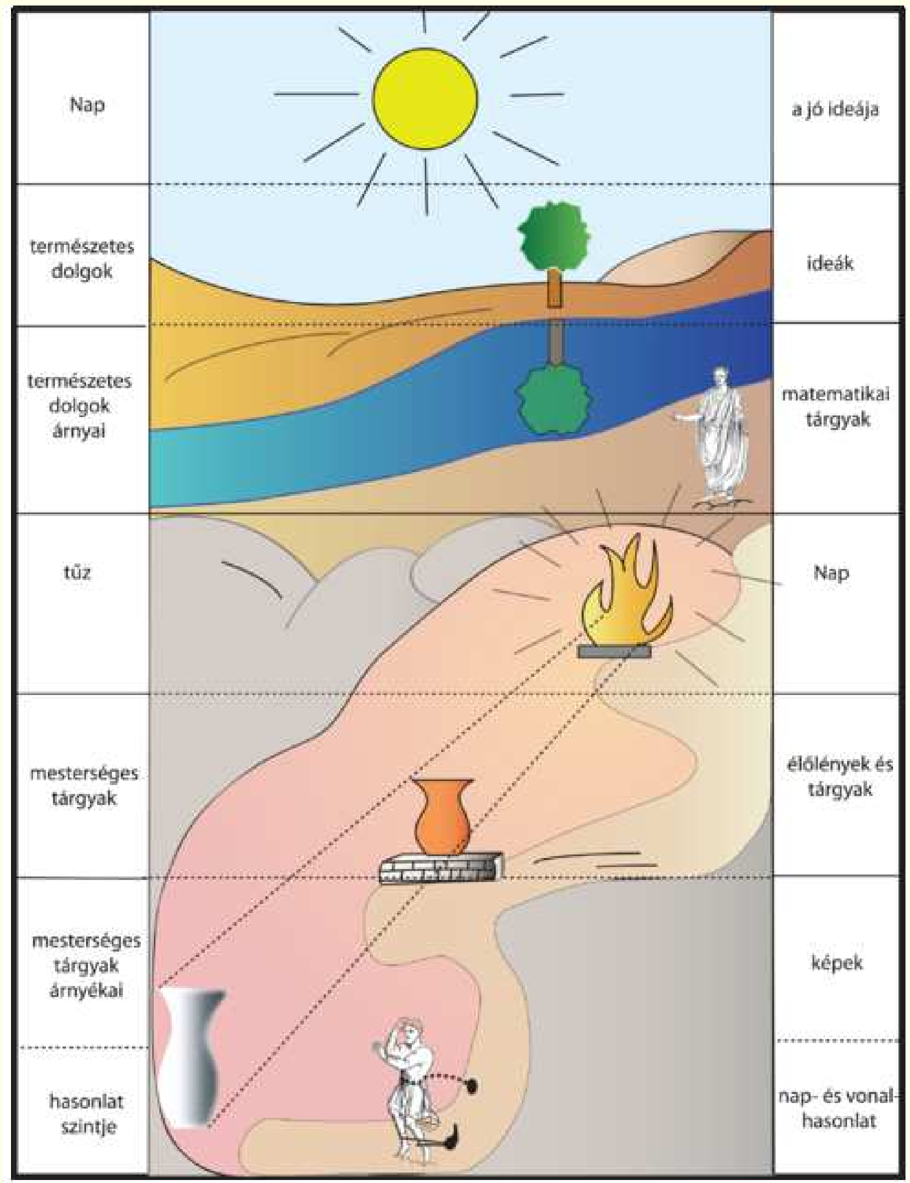




Hány dimenziós a világunk? Hogyan lehetne a dimenziót megmérni?

Barlanghasonlat

Már a görögök is gondolkodtak a problémán: Platón



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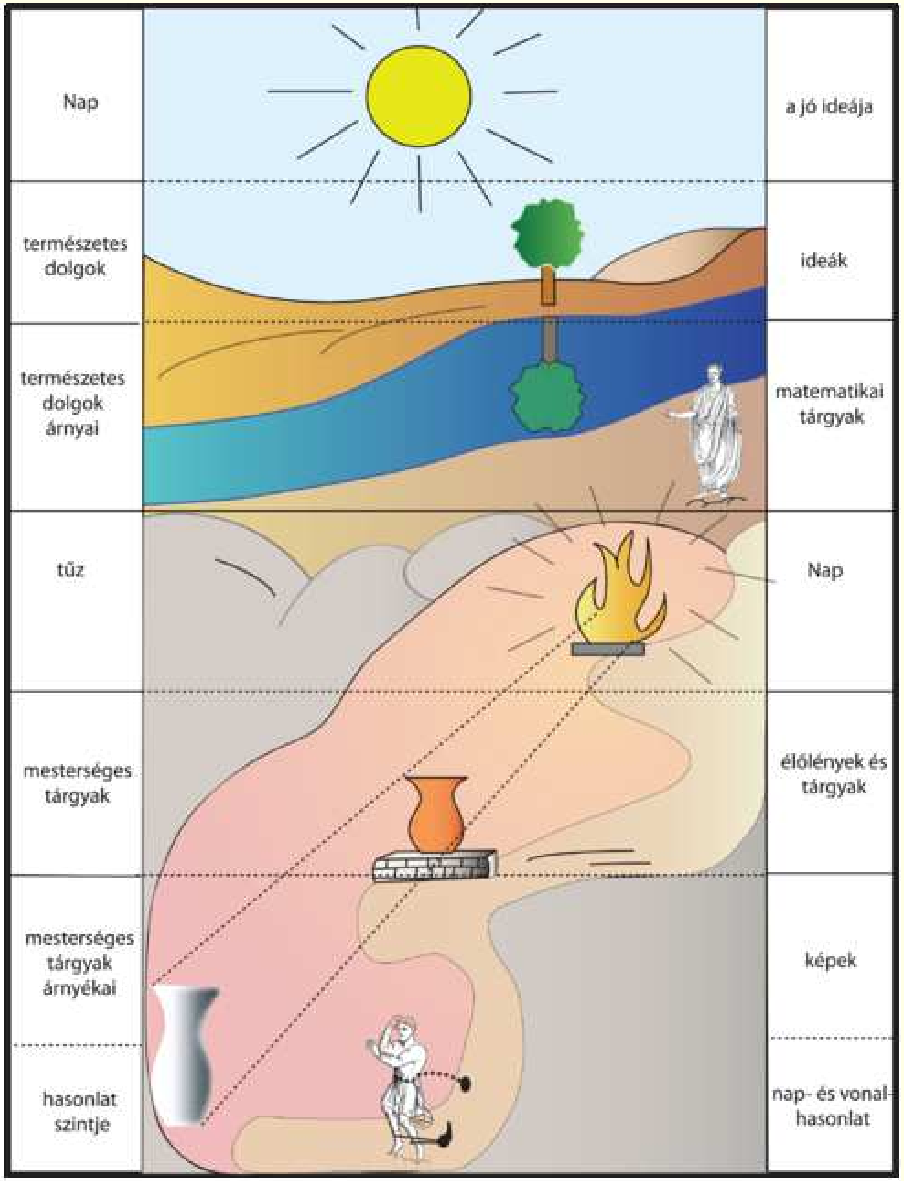


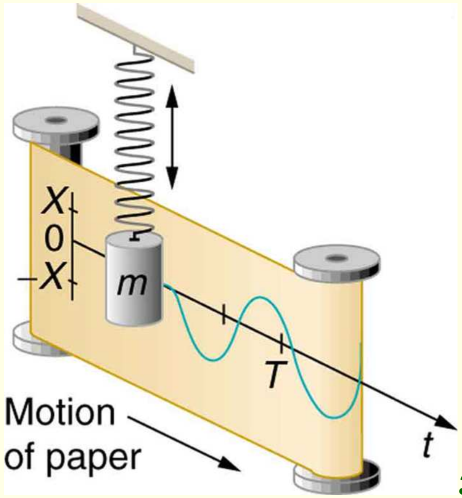
Mi van ha a háromdimenziós világunk sem az igazi, van egy magasabb dimenziós világ, aminek, mi csak az árnyékai vagyunk.

Hogyan tudnánk fizikai kísérletekkel kimutatni az extra dimenziót?

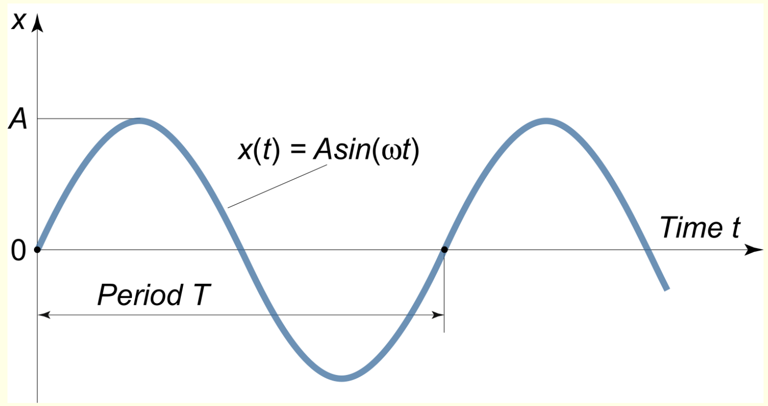
Figyelmeztetés!

Figyelmeztetés!

Harmonikus rezgés



$x(t) = A\sin(\omega t)$



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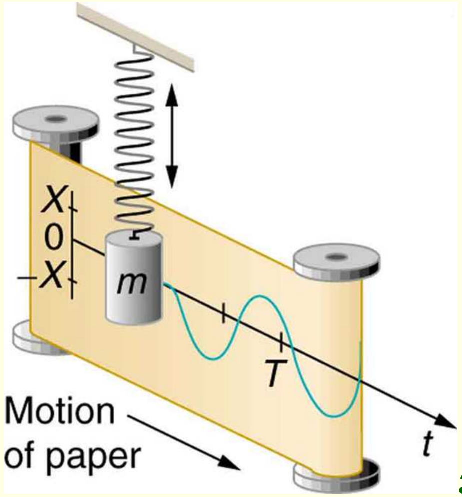

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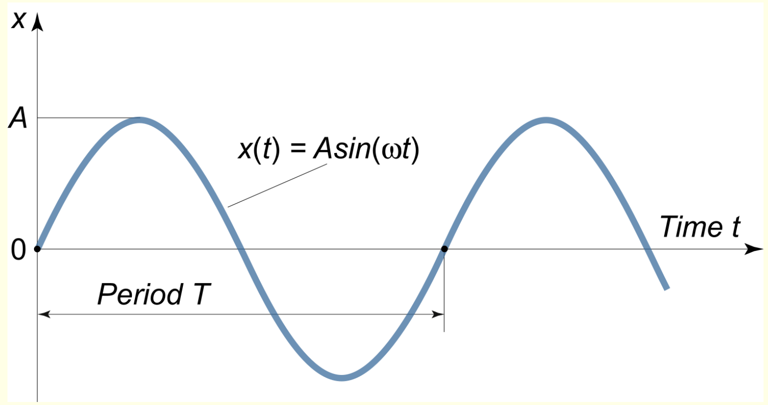
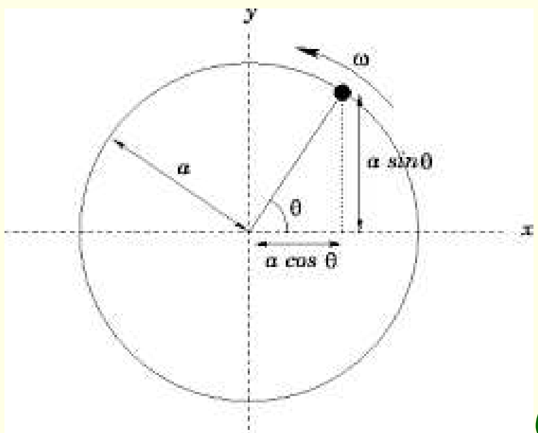
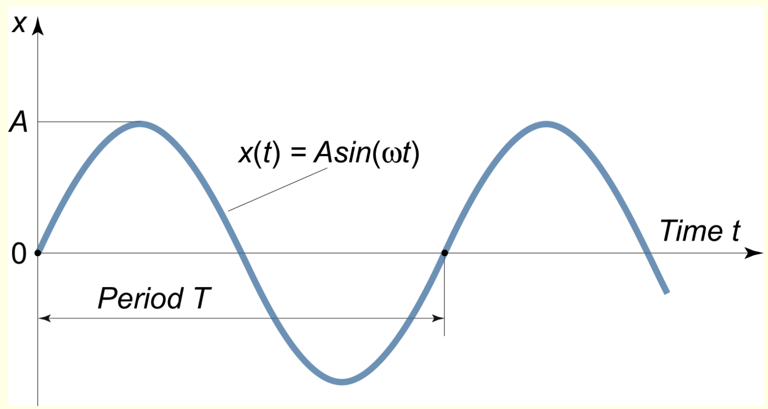


Egyenletes körmozgás és vetülete



$\theta = \omega t$



Egyenletes körmozgás a barlang falán harmonikus rezgőmozgásnak látszik! Ezek ekvivalens, megkülönböztethetlen leírások. (Habár az egyik egyszerűbb, mint a másik.)

Bolygómozgás

Planetárium vagy háromdimenziós mozgás?

Mars mozgása

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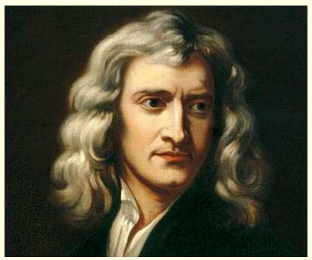
Planetárium vagy háromdimenziós mozgás?

Mars mozgása



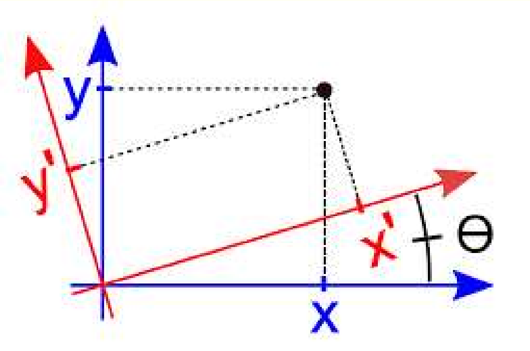
Newton: mozgásegyenlet

$$F_x = ma_x = m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}$$
$$F_y = ma_y = m\frac{d^2y}{dt^2} = -\frac{\partial V}{\partial y}$$
$$F_z = ma_z = m\frac{d^2z}{dt^2} = -\frac{\partial V}{\partial z}$$

gravitációs potenciál

$$V(r) = -\gamma\frac{mM}{r} = -\gamma\frac{mM}{\sqrt{x^2+y^2+z^2}}$$

Dimenzió az egyenletek alakjából



Forgassuk el a koordináta rendszerünket $z' = z$ körül

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta\, x + \sin\theta\, y \\ -\sin\theta\, x + \cos\theta\, y \end{pmatrix} \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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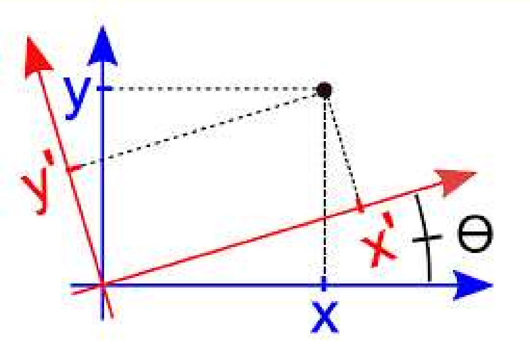

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A mozgásegyenlet alakja nem változik $\begin{pmatrix} m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x} \\ m\frac{d^2y}{dt^2} = -\frac{\partial V}{\partial y} \\ m\frac{d^2z}{dt^2} = -\frac{\partial V}{\partial z} \end{pmatrix} \rightarrow \begin{pmatrix} m\frac{d^2x'}{dt^2} = -\frac{\partial V}{\partial x'} \\ m\frac{d^2y'}{dt^2} = -\frac{\partial V}{\partial y'} \\ m\frac{d^2z'}{dt^2} = -\frac{\partial V}{\partial z'} \end{pmatrix}$

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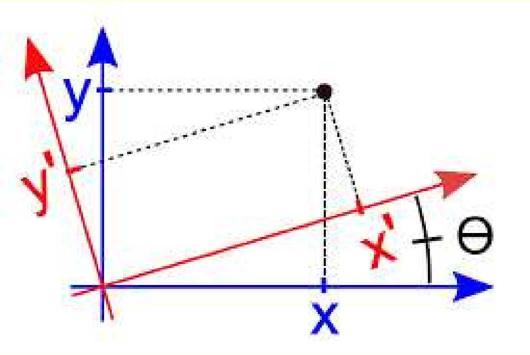

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A kovariancia transzformációk a háromdimenziós tér hossztartó lináris leképezései: $SO(3)$ (komplex tér $SU(3)$)

$$O_z = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, O_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}, O_y = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$$

A Newton egyenlet 3d jelölésben: $m\frac{dx_i^2}{dt^2} = F_i = -\frac{\partial V}{\partial x_i}$ $\quad i = 1, 2, 3$
$(x_1, x_2, x_3) = (x, y, z)$

Egyenletek egyszerűsége

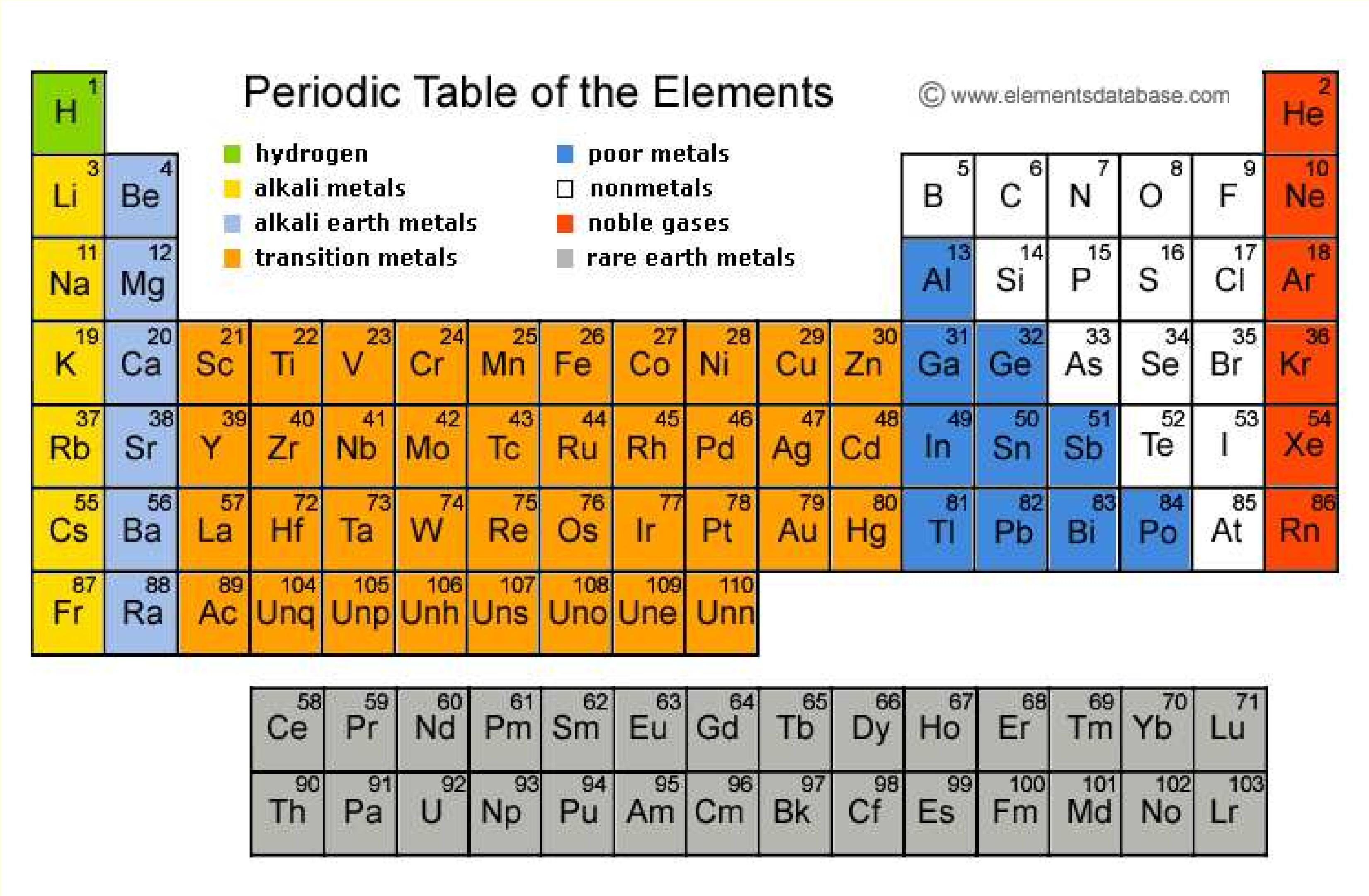
Forgó koordináta rendszerben

$$
\begin{aligned}
\boldsymbol{a} = \frac{\mathrm{d}^2 \boldsymbol{r}}{\mathrm{d}t^2} &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\left[\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right] + \boldsymbol{\omega}\times\boldsymbol{r}\right) \\
&= \left[\frac{\mathrm{d}^2 \boldsymbol{r}}{\mathrm{d}t^2}\right] + \boldsymbol{\omega}\times\left[\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right] + \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}\times\boldsymbol{r} + \boldsymbol{\omega}\times\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \\
&= \left[\frac{\mathrm{d}^2 \boldsymbol{r}}{\mathrm{d}t^2}\right] + \boldsymbol{\omega}\times\left[\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right] + \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}\times\boldsymbol{r} + \boldsymbol{\omega}\times\left(\left[\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right] + \boldsymbol{\omega}\times\boldsymbol{r}\right) \\
&= \left[\frac{\mathrm{d}^2 \boldsymbol{r}}{\mathrm{d}t^2}\right] + \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}\times\boldsymbol{r} + 2\boldsymbol{\omega}\times\left[\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right] + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})\,.
\end{aligned}
$$

Polár koordinátákban

$$
\begin{aligned}
&\mathbf{r} = \mathbf{r}(t) = r\hat{\mathbf{e}}_r \\
&\mathbf{v} = v\hat{\mathbf{e}}_r + r\frac{d\theta}{dt}\hat{\mathbf{e}}_\theta + r\frac{d\varphi}{dt}\sin\theta\,\hat{\mathbf{e}}_\varphi \\
&\mathbf{a} = \left(a - r\left(\frac{d\theta}{dt}\right)^2 - r\left(\frac{d\varphi}{dt}\right)^2\sin^2\theta\right)\hat{\mathbf{e}}_r \\
&\quad + \left(r\frac{d^2\theta}{dt^2} + 2v\frac{d\theta}{dt} - r\left(\frac{d\varphi}{dt}\right)^2\sin\theta\cos\theta\right)\hat{\mathbf{e}}_\theta \\
&\quad + \left(r\frac{d^2\varphi}{dt^2}\sin\theta + 2v\frac{d\varphi}{dt}\sin\theta + 2r\frac{d\theta}{dt}\frac{d\varphi}{dt}\cos\theta\right)\hat{\mathbf{e}}_\varphi
\end{aligned}
$$

Az elemek periódusos rendszere



Minek az árnyéka a periódusos rendszer?

Dimenzió leolvasása a spektrumból

Elektromos kölcsönhatás (potenciális energia $V(r) = k\frac{Zq}{r}$)

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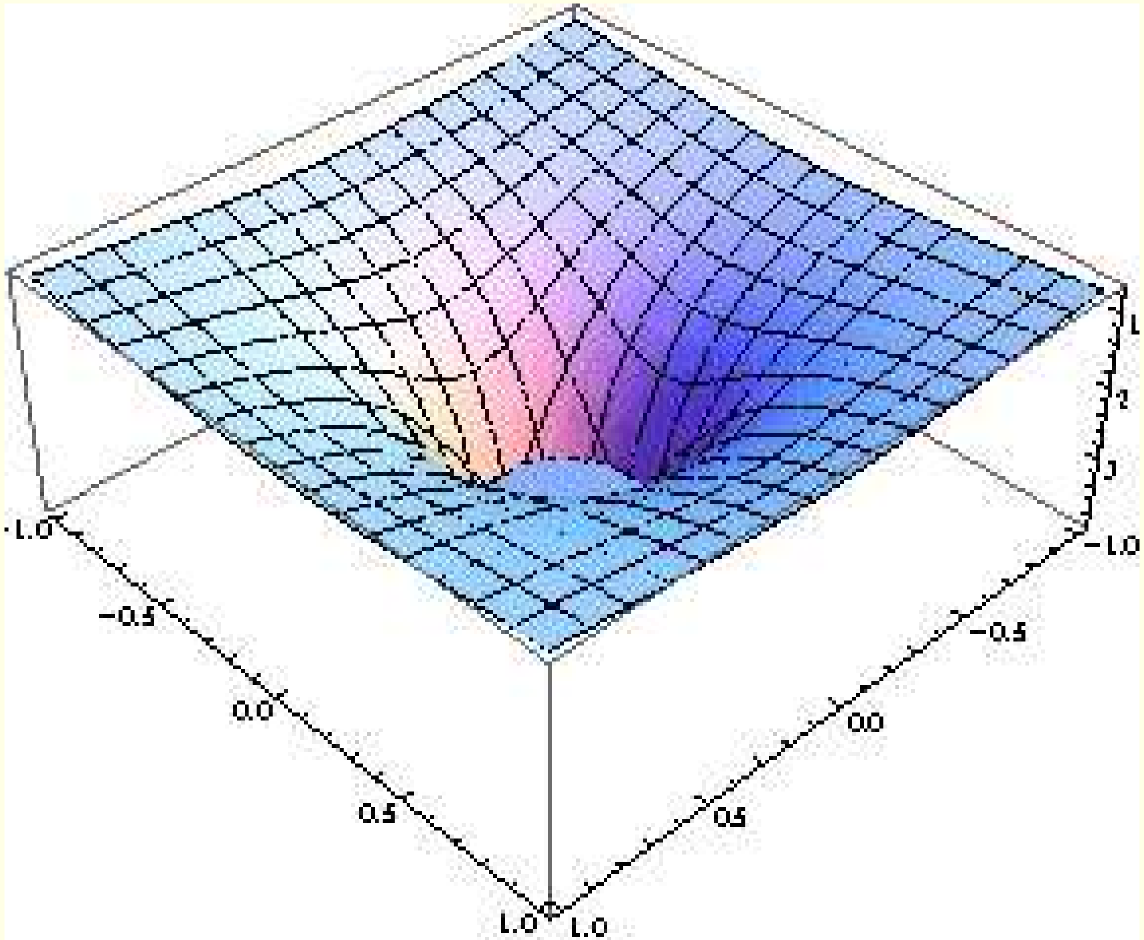

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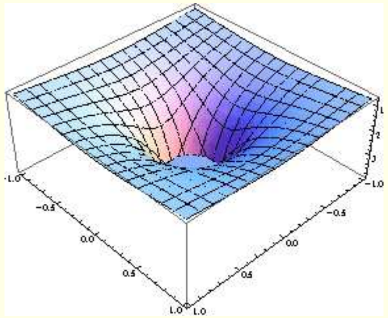

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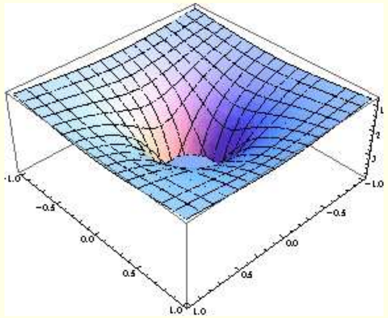

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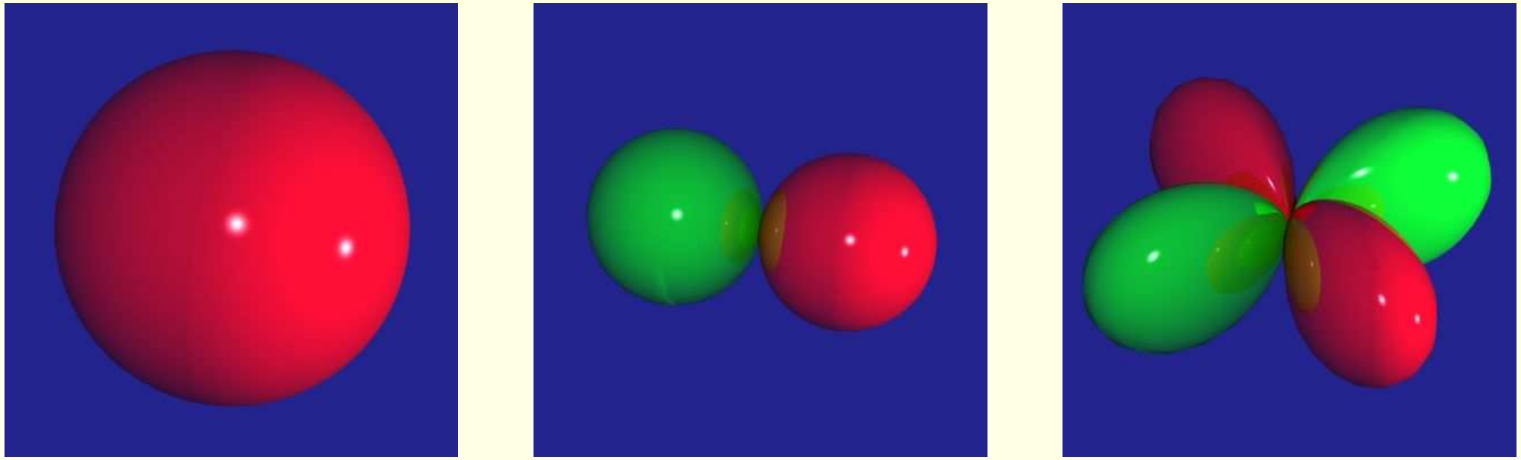

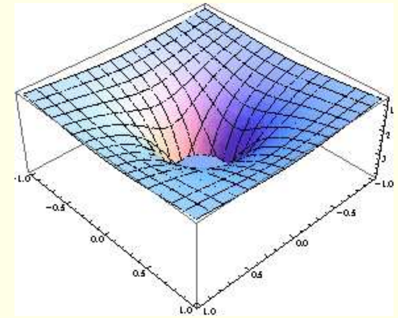

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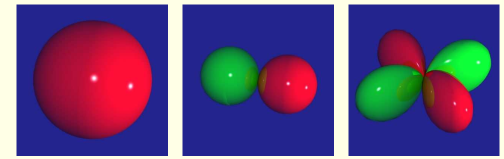

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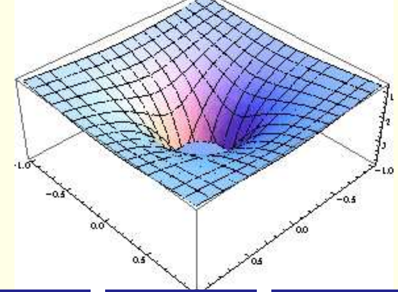

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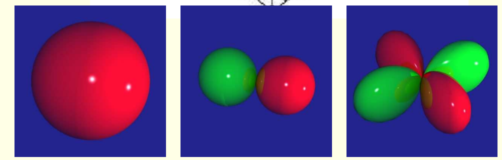
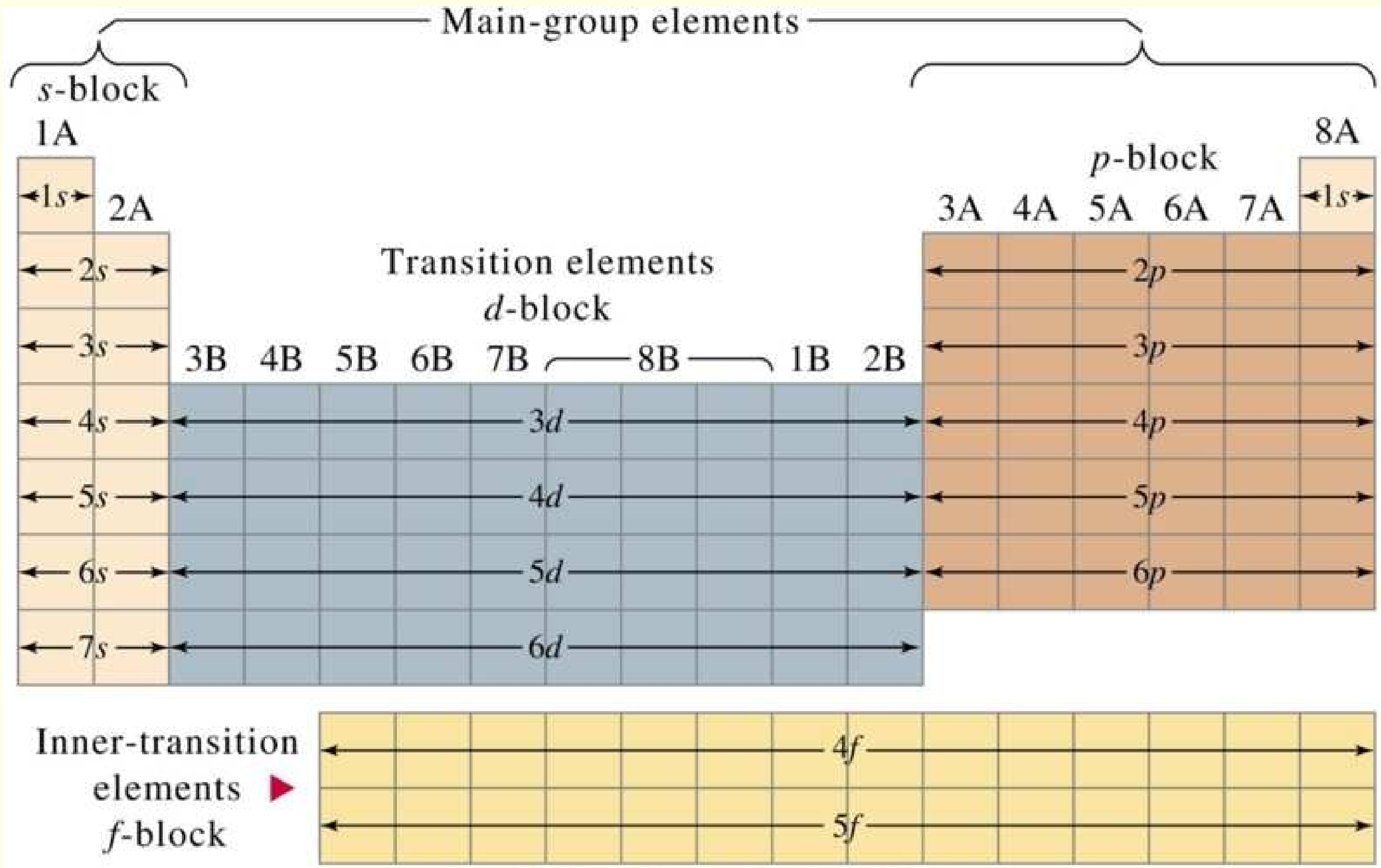
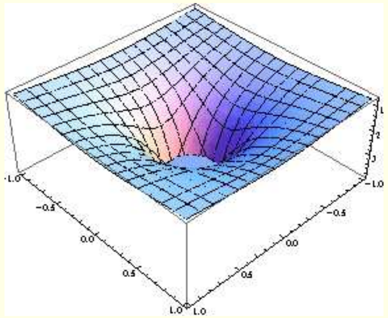




Dimenzió leolvasása a spektrumból



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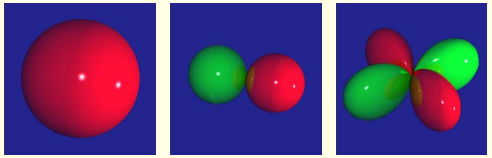

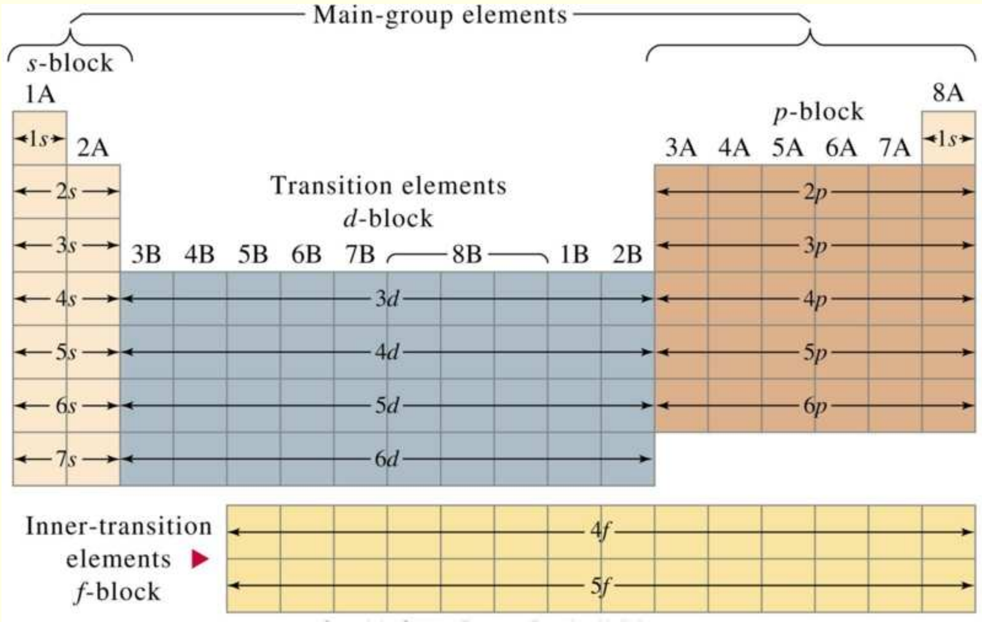

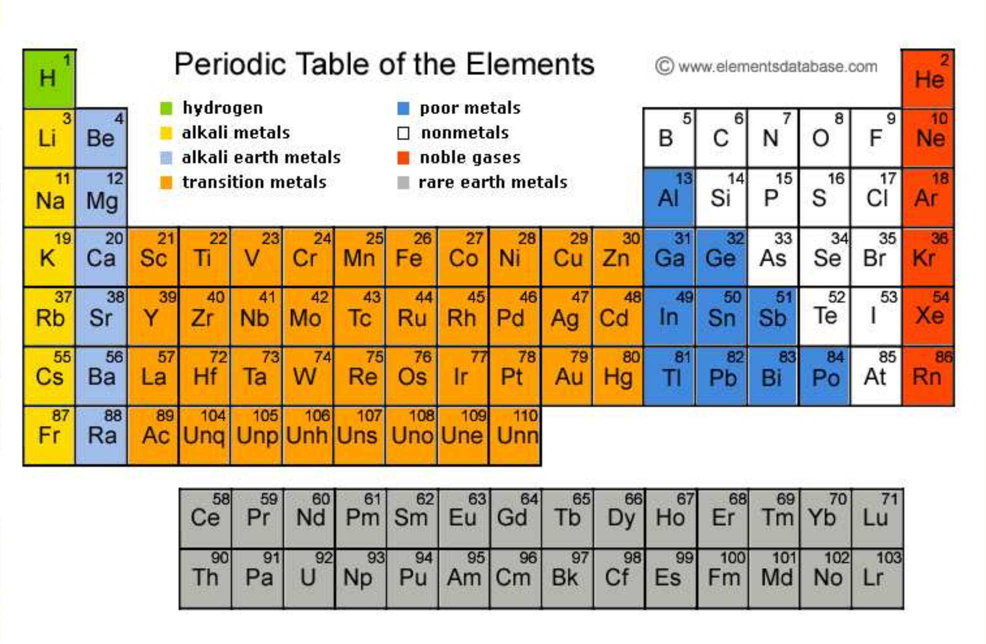

Dimenzió: p-elemek = 2×dimenzió

Elektromos és mágneses kölcsönhatás egyesítése

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

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Maxwell egyenletek:

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \qquad \nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

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Kovariancia transzformációk: forgatások + Lorentz transzformációk

$$x' = \frac{(x+vt)}{\sqrt{1-\frac{v^2}{c^2}}}; t' = \frac{(t+\frac{xv}{c^2})}{\sqrt{1-\frac{v^2}{c^2}}} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

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Az idő a negyedik dimenzió $(ct, x, y, z) = x^\mu \qquad \mu = 1, 2, 3, 4$

$$\begin{pmatrix} \gamma & \frac{v\gamma}{c} & 0 & 0 \\ \frac{vc}{\gamma} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};\; \begin{pmatrix} \gamma & 0 & \frac{v\gamma}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{vc}{\gamma} & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},\; \begin{pmatrix} \gamma & 0 & 0 & \frac{v\gamma}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{vc}{\gamma} & 0 & 0 & \gamma \end{pmatrix} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

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Az idő a negyedik dimenzió $(ct, x, y, z) = x^\mu \qquad \mu = 1, 2, 3, 4$

$$\begin{pmatrix} \gamma & \frac{v\gamma}{c} & 0 & 0 \\ \frac{vc}{\gamma} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \begin{pmatrix} \gamma & 0 & \frac{v\gamma}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{vc}{\gamma} & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} \gamma & 0 & 0 & \frac{v\gamma}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{vc}{\gamma} & 0 & 0 & \gamma \end{pmatrix} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

Maxwell egyenletek ha $(c\rho, J_x, J_y, J_z) = J^\mu$ akkor $\partial_\mu F^{\mu\nu} = J^\nu$

$\partial^\mu F_{\mu\nu}(E \to B, B \to -E) = 0 \to F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ potenciál (ϕ, A_x, A_y, A_z)

Melyik a fizika legszebb egyenlete?

Physics World, közvéleménykutatás:

Melyik a fizika legszebb egyenlete?

Physics World, közvéleménykutatás:

Leonhard Euler (1707-1783)

$e^{i\cdot\pi} + 1 = 0$

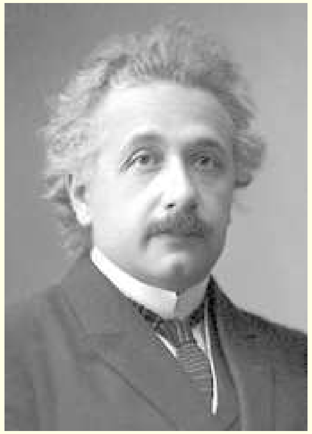
$i, \pi, e, 1, 0$ and $+, \cdot, \hat{}$

James Clerk Maxwell (1831-1879)

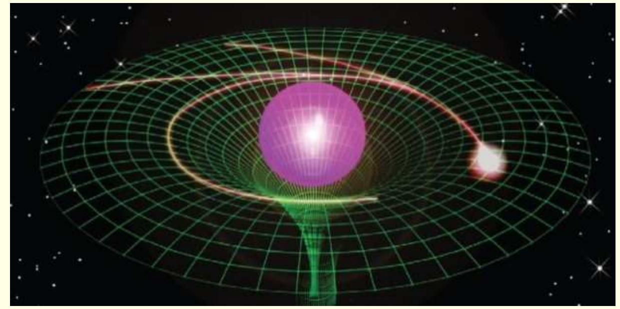
$d \star F = j \quad ; \quad dF = 0$

egyesítés: elektromágnesség

Gravitáció + relativitás elmélet



$$\begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ \cdot & g_{xx} & g_{xy} & g_{xz} \\ \cdot & \cdot & g_{yy} & g_{yz} \\ \cdot & \cdot & \cdot & g_{zz} \end{pmatrix}$$



metrikus tenzor $v \cdot u = g_{\mu\nu} u^\mu v^\nu$

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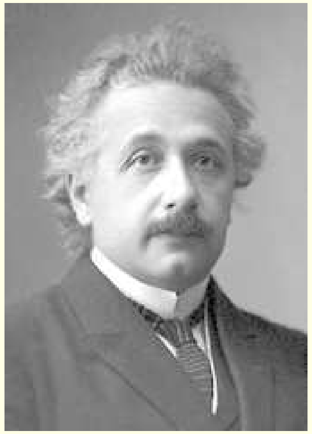

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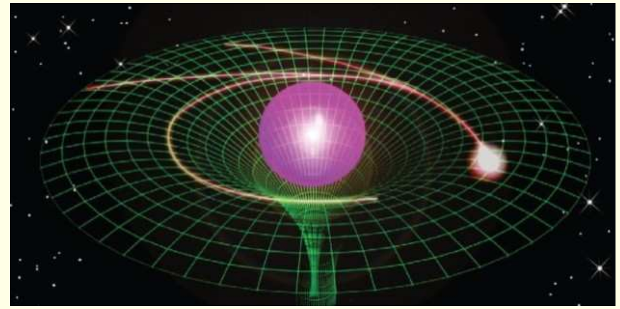

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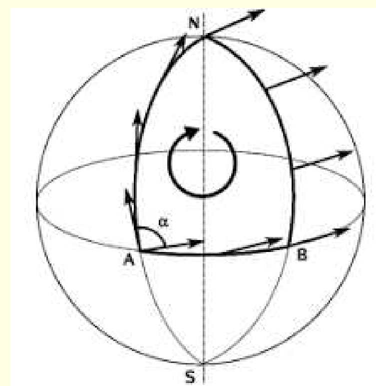
metrika $\rightarrow$ térgörbület a párhuzamos eltolásokból



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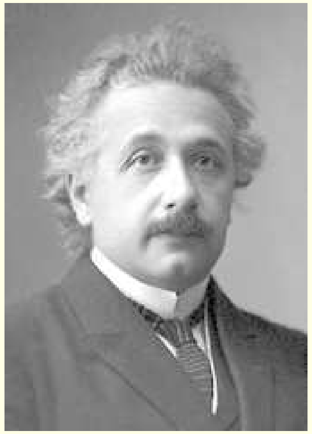

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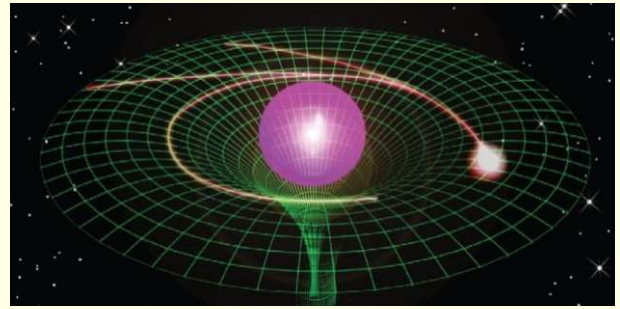

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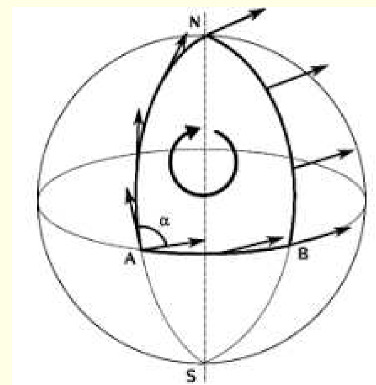
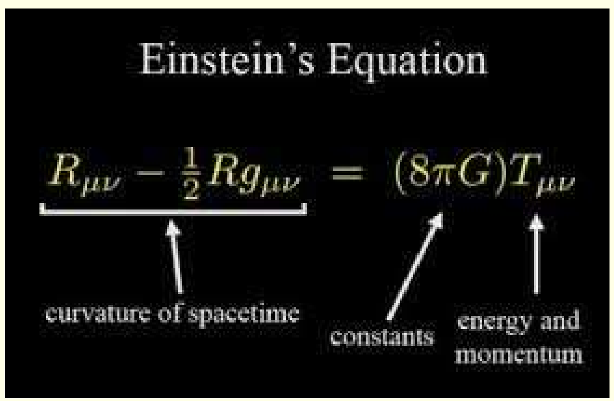


Einstein egyenlet

görbület=anyag



Gravitációs és elektromágneses kölcsönhatás egyesítése

Mindkettőben $V(r) = \frac{\alpha}{r}$ potenciál

DE: gravitáció = általános relativitás elmélet

$$\begin{pmatrix} g_{55} & g_{5t} & g_{5x} & g_{5y} & g_{5z} \\ \cdot & g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ \cdot & \cdot & g_{xx} & g_{xy} & g_{xz} \\ \cdot & \cdot & \cdot & g_{yy} & g_{yz} \\ \cdot & \cdot & \cdot & \cdot & g_{zz} \end{pmatrix}$$

Kaluza: 4+1 dimenziós gravitáció

$g_{5\mu} = (\phi, A_x, A_y, A_z)$ görbülete $F_{\mu\nu}$

Gravitációs és elektromágneses kölcsönhatás egyesítése

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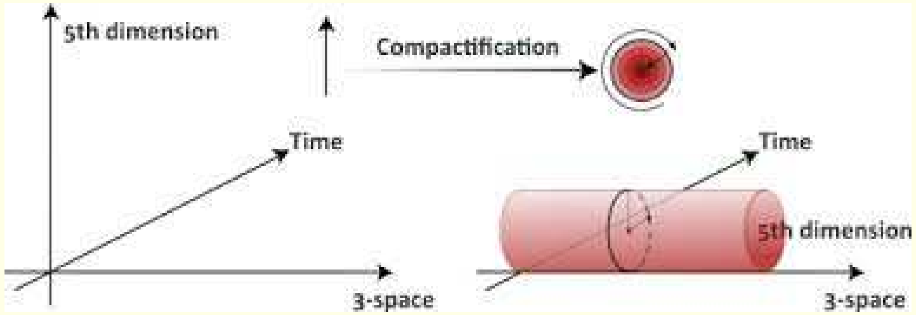
4+1 D Einstein egyenletek = 3+1 D Einstein + Maxwell egyenletek

De miért nem látjuk?
Klein: Mert fel van csavarodva.



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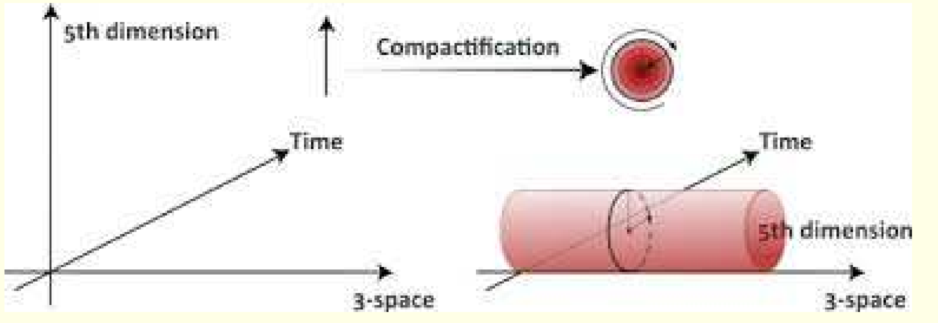
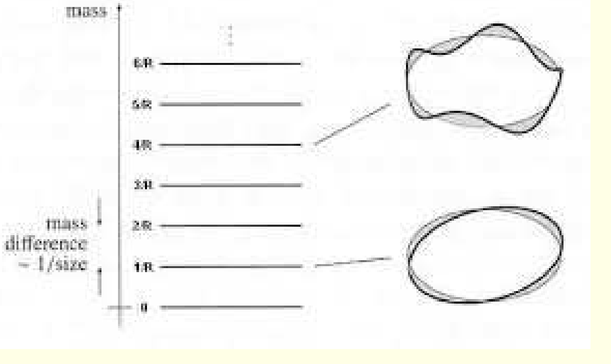


Kvantumelmélet
Impulzus kvantálás $p = \frac{n}{R}$,
energiaspektrum $E_n = \frac{n}{R}$ egyenközű

De mi van a természetben?



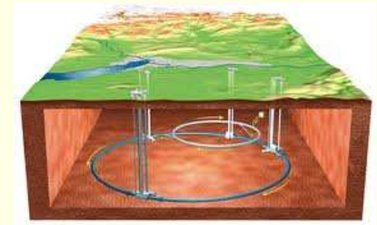
Miből áll a világ és hogyan hat kölcsön?

Miből áll a világ és hogyan hat kölcsön?

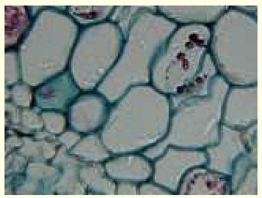
mikroszkóp

elektronmikroszkóp

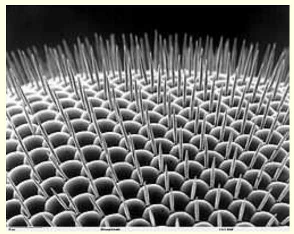
szinkrotron



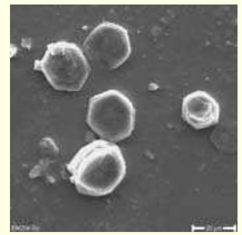
LHC



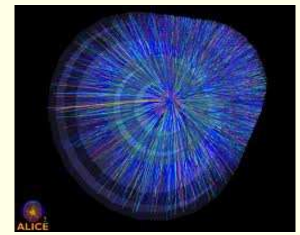
mikrométer



nanométer



sok atom



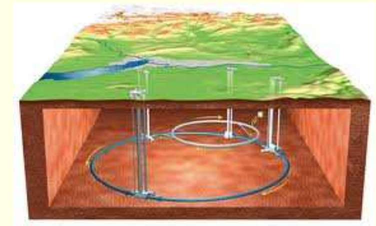
kvarkok,leptonok

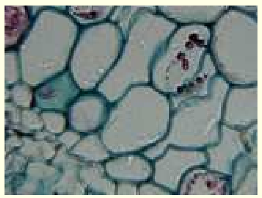
Miből áll a világ és hogyan hat kölcsön?


mikroszkóp


elektronmikroszkóp


szinkrotron


LHC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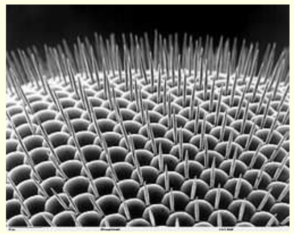
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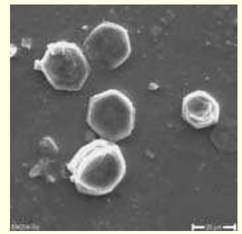
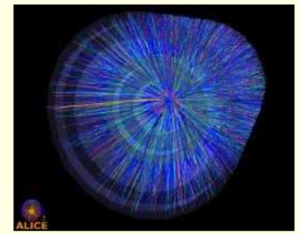
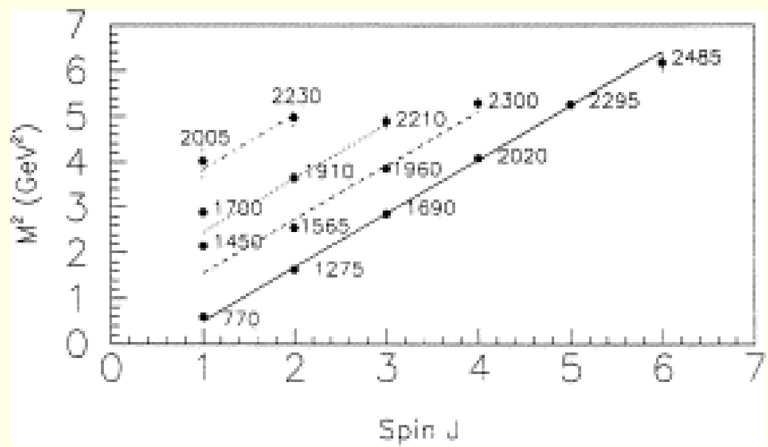

sok atom


kvarkok,leptonok

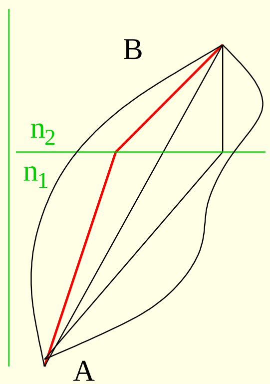
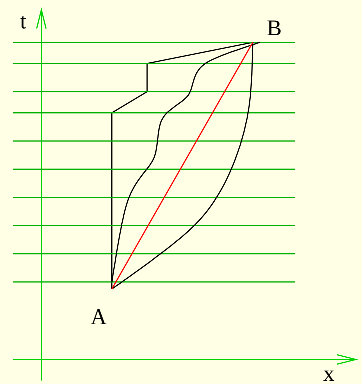
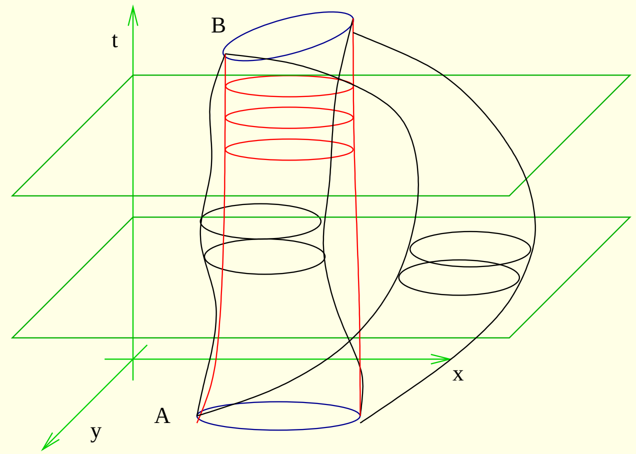
hadron spektrum

Jó közelítéssel $M^2(J) = \alpha' J + \beta n$

DE NEM $M(n) = \frac{n}{R}$



Húrok klasszikus dinamikája

<table>
<tr><td>fény</td><td>pontrészecske</td><td>húr</td></tr>
<tr><td>Fermat elv</td><td>téridő (x, t)</td><td>téridő (x, y, t)</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>idő minimális</td><td>téridő út minimális</td><td>téridő felület minimális</td></tr>
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Kvantum húrok spektruma

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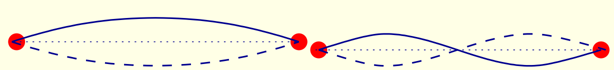
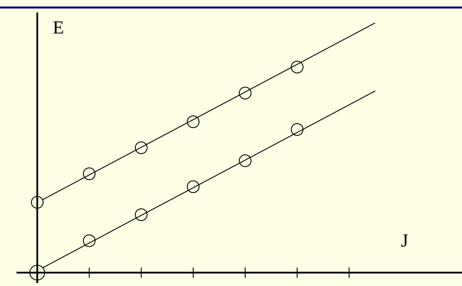
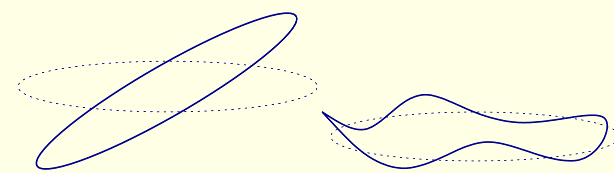
<tr><td></td><td>
</td><td></td></tr>
<tr><td>foton, elektron, kvarkok+...</td><td>dimenzió = 10</td><td>graviton+...</td></tr>
<tr><td>mértékelmélet anyaggal</td><td>$M^2(J) = \alpha' J + \text{const.}$</td><td>gravitáció</td></tr>
</table>

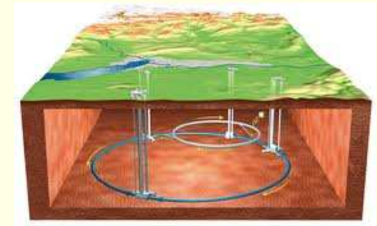
Hogyan magyarázható a hadronspektrum?

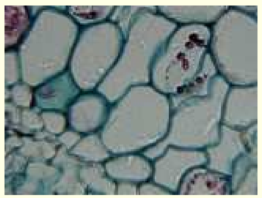
Hogyan magyarázható a hadronspektrum?

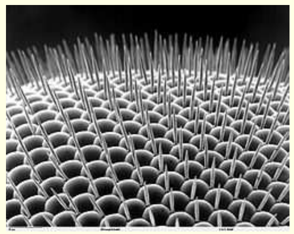

mikroszkóp

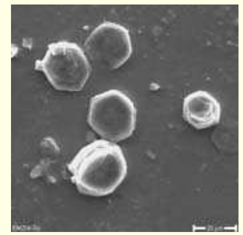

elektronmikroszkóp

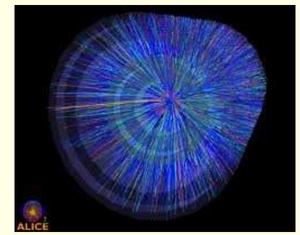

szinkrotron


LHC


mikrométer


nanométer


sok atom



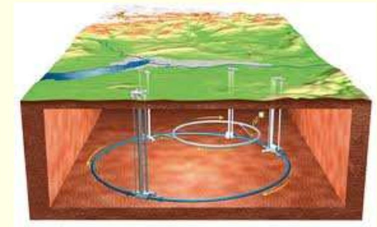
kvarkok,leptonok

Hogyan magyarázható a hadronspektrum?

mikroszkóp

elektronmikroszkóp

szinkrotron



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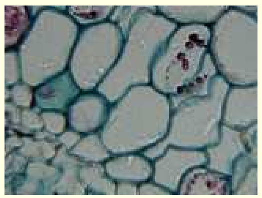
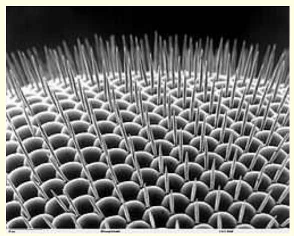


mikrométer



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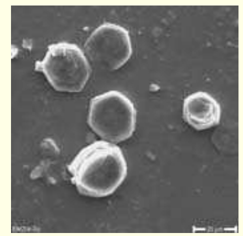

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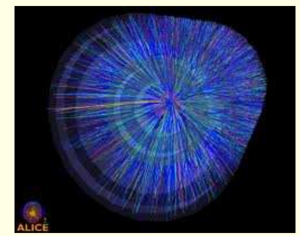
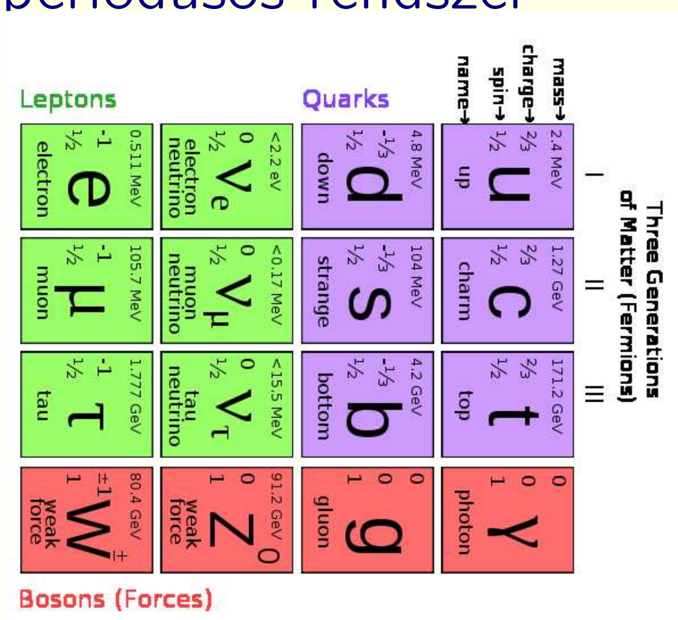
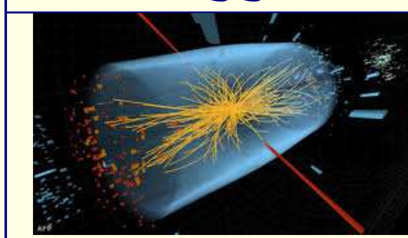


kvarkok,leptonok

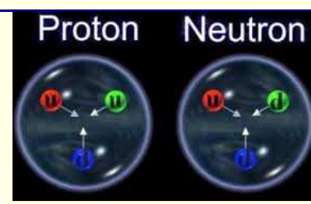
periódusos rendszer



Higgs

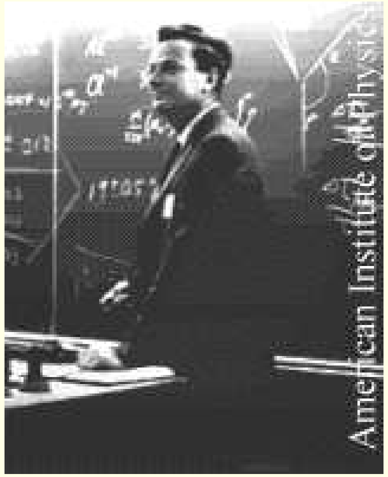


anyag



	Kölcsönhatás
γ	elektromágneses
$W^{\pm}, Z$	gyenge
g	erős
gr	gravitációs

Kvantumelektrodinamika



Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

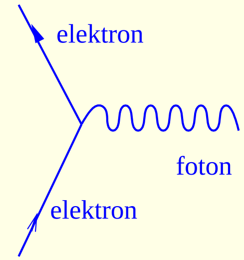
Kvantumos

mértékelmélet

Elektromos + mágneses kh: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

+Kvantumelmélet→kvantumelektrodinamika

$U(1)$ mértékelmélet: $A_\mu \to A_\mu + \partial_\mu \Lambda$



$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - e\bar{\Psi}A\!\!\!/\Psi$$

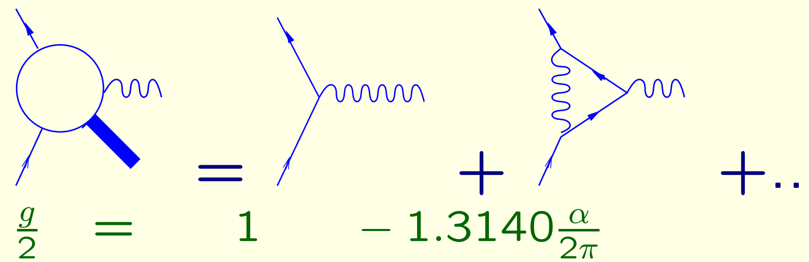
kísérleti eredmény: $\underline{\mu} = g\frac{e\hbar}{2mc}\underline{s}$ ahol $g = 2(1+a)$

[Gabrielse 2006]: $a = 1159652180.85(.76) \times 10^{-12}$

perturbáció számítás:

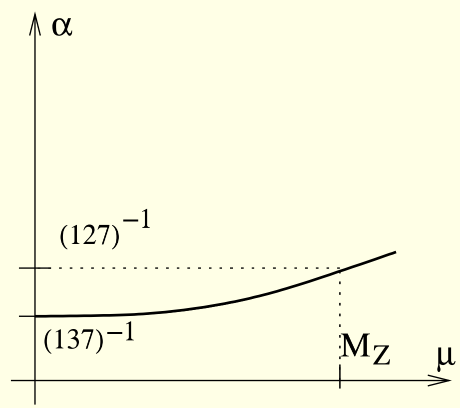
Feynman gráfok

$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$

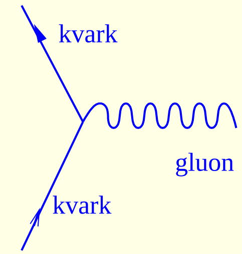


impulzusfüggő csatolás:

$\beta(\alpha) = \mu\frac{\partial\alpha}{\partial\mu} > 0$



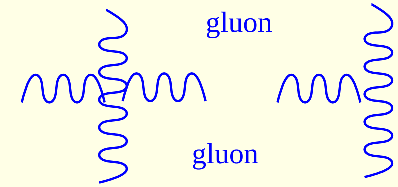
Kvantumszíndinamika



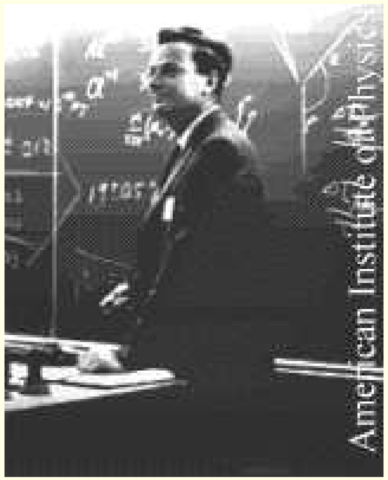
foton $A_\mu \leftrightarrow G_\mu^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$

elektron $\Psi_e \leftrightarrow \Psi_{kvark}$ kvark

$SU(3)$ mértékelmélet: $G_\mu \rightarrow g^{-1} G_\mu g + g^{-1} \partial_\mu g$

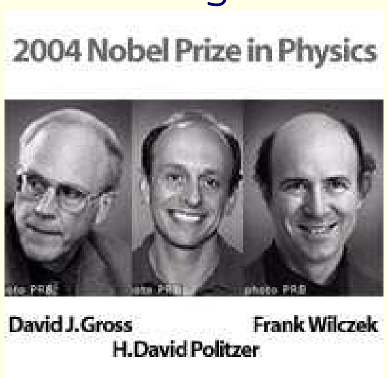


$$\mathcal{L} = -\frac{1}{4} F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\bar{\Psi}G\!\!\!/\Psi$$



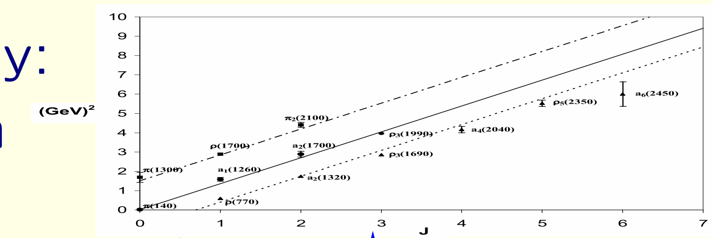
Kvantum-mértékelmélet

aszimptotikus szabadság



kísérleti eredmény:

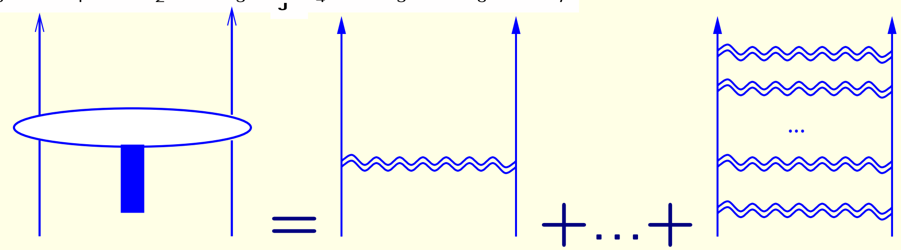
hadron spektrum



perturbáció számítás:

Feynman gráfok

$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$

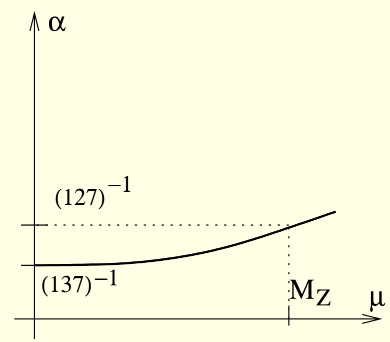


impulzusfüggő csatolás:

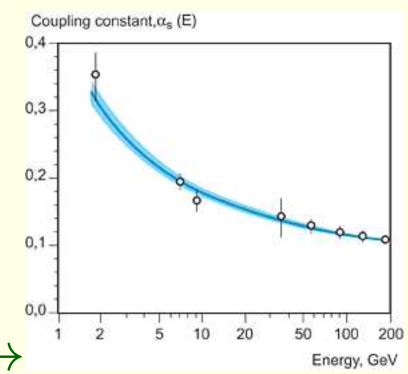
$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$

aszimptotikus szabadság

bezárás



$\leftrightarrow$



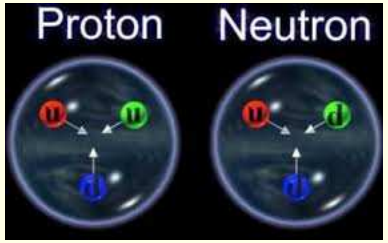
Erős kölcsönhatás=kvantumszíndinamika

SU(3) mértékelmélet

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\bar{\Psi}G\!\!\!/\Psi$$

Wigner Jenő:
The simplicities of natural laws arise
through the complexities of the language
we use for their expression.

Alacsony energia: nemperturbatív fizika
bezárás
hadronspektrum,
nehézion ütközés



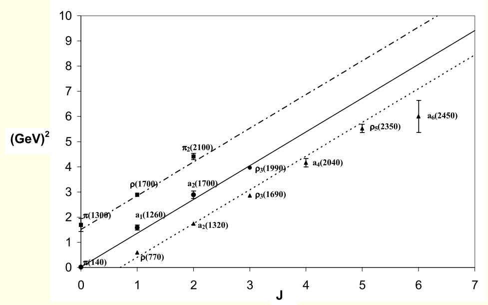
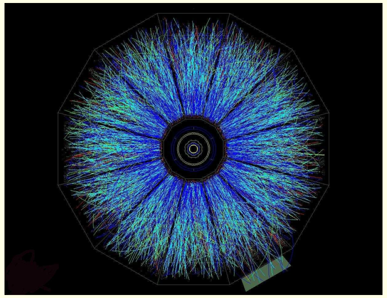
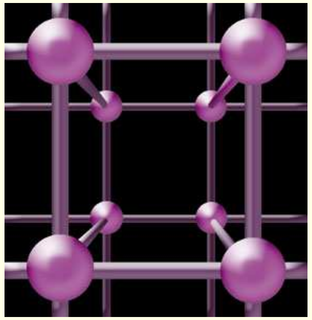


Fig. 1.



Rács-mértékelmélet:

világ 32^4

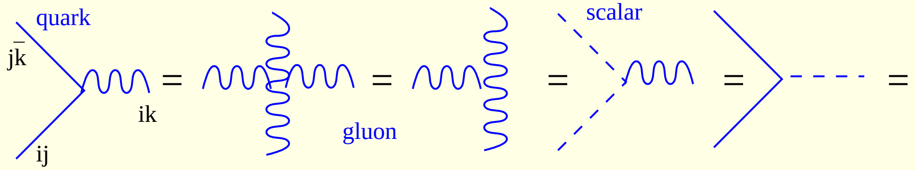


protontömeg $\surd$
nehézionütközés?

Wigner: It is nice to know that the computer understands the problem. But I would like to understand it too.

egyszerűsített modell

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM



$$\frac{2}{g^2_{YM}} \int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}D\!\!\!/\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

A részecskefizika standard modellje

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Dirac} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm gauge/\psi} \; . \tag{1}$$

Here,

$$\mathcal{L}_{\rm Dirac} = i\bar{e}^i_{\rm L}\not\partial e^i_{\rm L} + i\bar{\nu}^i_{\rm L}\not\partial \nu^i_{\rm L} + i\bar{e}^i_{\rm R}\not\partial e^i_{\rm R} + i\bar{u}^i_{\rm L}\not\partial u^i_{\rm L} + i\bar{d}^i_{\rm L}\not\partial d^i_{\rm L} + i\bar{u}^i_{\rm R}\not\partial u^i_{\rm R} + i\bar{d}^i_{\rm R}\not\partial d^i_{\rm R} \; ; \tag{2}$$

$$\mathcal{L}_{\rm mass} = -v\left(\lambda^i_e \bar{e}^i_{\rm L} e^i_{\rm R} + \lambda^i_u \bar{u}^i_{\rm L} u^i_{\rm R} + \lambda^i_d \bar{d}^i_{\rm L} d^i_{\rm R} + {\rm h.c.}\right) - M_W^2 W^+_\mu W^{-\mu} - \frac{M_W^2}{2\cos^2\theta_W} Z_\mu Z^\mu \; ; \tag{3}$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{WZA} \; , \tag{4}$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \; , \end{aligned} \tag{5}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = \; & i g_2 \cos\theta_{\rm W} \left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu Z^\nu + W^+_{\mu\nu}W^{-\mu}Z^\nu - W^-_{\mu\nu}W^{+\mu}Z^\nu\right] \\ & + ie\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu A^\nu + W^+_{\mu\nu}W^{-\mu}A^\nu - W^-_{\mu\nu}W^{+\mu}A^\nu\right] \\ & + g_2^2\cos^2\theta_{\rm W}\left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu} Z_\nu Z^\nu\right) \\ & + g_2^2\left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu} A_\nu A^\nu\right) \\ & + g_2 e\cos\theta_{\rm W}\left[W^+_\mu W^-_\nu\left(Z^\mu A^\nu + Z^\nu A^\mu\right) - 2W^+_\mu W^{-\mu} Z_\nu A^\nu\right] \\ & + \frac{1}{2}g_2^2\left(W^+_\mu W^-_\nu\right)\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right) \; ; \end{aligned} \tag{6}$$

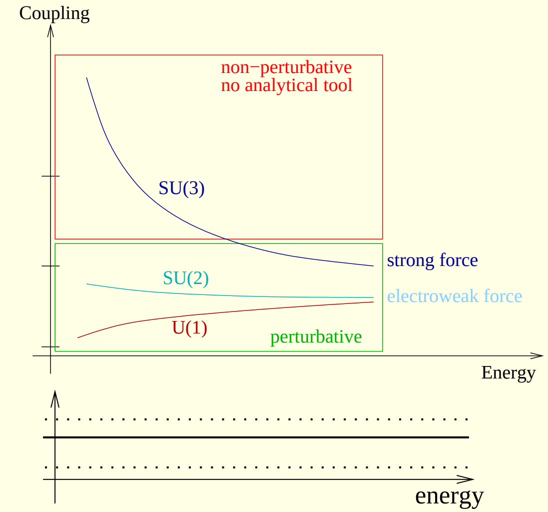
and

$$\mathcal{L}_{\rm gauge/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2\left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z_\mu J^\mu_Z\right) - eA_\mu J^\mu_A \; , \tag{7}$$

where

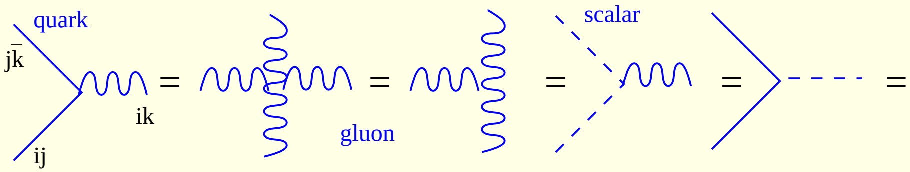
$$\begin{aligned} J^{\mu a}_{(3)} &= \bar{u}^i\gamma^\mu T^a_{(3)} u^i + \bar{d}^i\gamma^\mu T^a_{(3)} d^i \\ J^\mu_{W^+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}^i_{\rm L}\gamma^\mu e^i_{\rm L} + V^{ij}\bar{u}^i_{\rm L}\gamma^\mu d^j_{\rm L}\right) \\ J^\mu_{W^-} &= (J^\mu_{W^+})^* \\ J^\mu_Z &= \frac{1}{\cos\theta_{\rm W}}\left[\frac{1}{2}\bar{\nu}^i_{\rm L}\gamma^\mu\nu^i_{\rm L} + \left(-\frac{1}{2}+\sin^2\theta_{\rm W}\right)\bar{e}^i_{\rm L}\gamma^\mu e^i_{\rm L} + (\sin^2\theta_{\rm W})\bar{e}^i_{\rm R}\gamma^\mu e^i_{\rm R}\right. \\ & \quad + \left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm L}\gamma^\mu u^i_{\rm L} + \left(-\frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm R}\gamma^\mu u^i_{\rm R} \\ & \quad \left. + \left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm L}\gamma^\mu d^i_{\rm L} + \left(\frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm R}\gamma^\mu d^i_{\rm R}\right] \\ J^\mu_A &= (-1)\bar{e}^i\gamma^\mu e^i + \left(\frac{2}{3}\right)\bar{u}^i\gamma^\mu u^i + \left(-\frac{1}{3}\right)\bar{d}^i\gamma^\mu d^i \; . \end{aligned} \tag{8}$$

Maximálisan (szuper)szimmetrikus mértékelmélet



Alapvető kölcsönhatások

kölcsönhatás	részecskék	mértékcsoport
elektromgáneses	foton+elektron	$U(1)$
elektrogyenge	$W^{\pm}, Z \quad \mu,\nu$+Higgs	$SU(2) \times U(1)$
erős	gluon+kvark	$SU(3)$
MaxSzim. SYM	gluon A_μ+kvark Ψ+skalar Φ	$SU(N)$



minden tér $N^2 - 1$ komponensű mátrix

$$
\begin{array}{ccccc}
Q_{1,2,3,4} & & \Psi^{ij}_{1,2,3,4} & & Q_{1,2,3,4} \\
& \nearrow & \circlearrowleft & \searrow & \\
A^{ij}_\mu & & su(4) = so(6) & \circlearrowleft & \Phi^{ij}_{1,2,3,4,5,6} \\
& \searrow & \circlearrowleft & \nearrow & \\
Q^\dagger_{1,2,3,4} & & \overline{\Psi}^{ij}_{1,2,3,4} & & Q^\dagger_{1,2,3,4}
\end{array}
$$

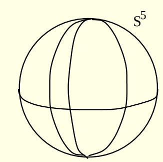
$$\mathcal{L} = \frac{2}{g^2_{YM}} \int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

Szimmetriák:

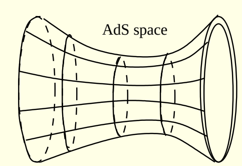
belső: $su(4) = so(6)$

5 dimenziós gömb



téridő: konform⊃Lorentz

5 dimenziós Anti-de Sitter tér



Maldacena: Mérték/gravitáció dualitás

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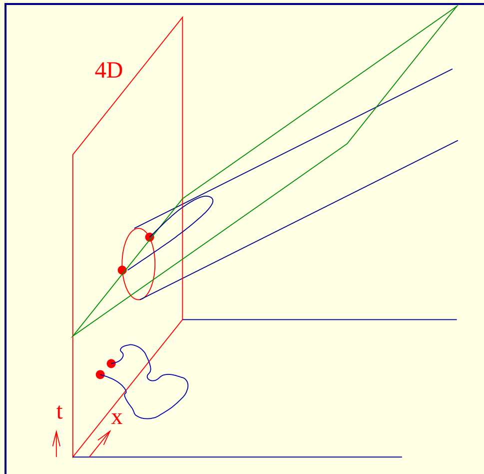
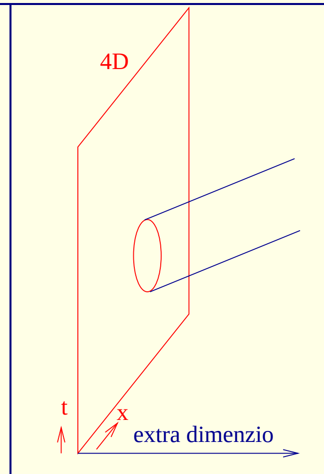
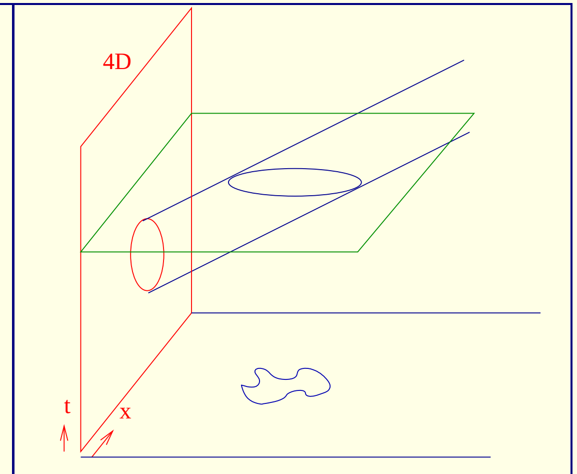
<tr><td>
</td><td>
</td><td>
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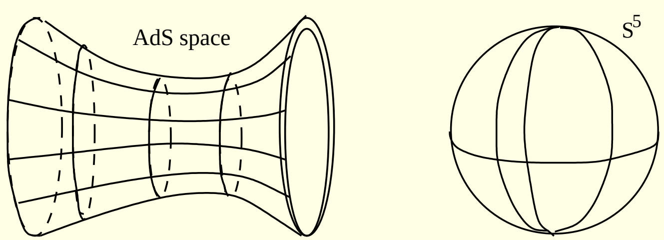
<tr><td>nyílt húr folyamat</td><td></td><td>zárt húr folyamat</td></tr>
<tr><td>mértékelmélet</td><td>=</td><td>gravitáció</td></tr>
</table>

$\mathcal{N}=4$ D=4 $SU(N)$ SYM

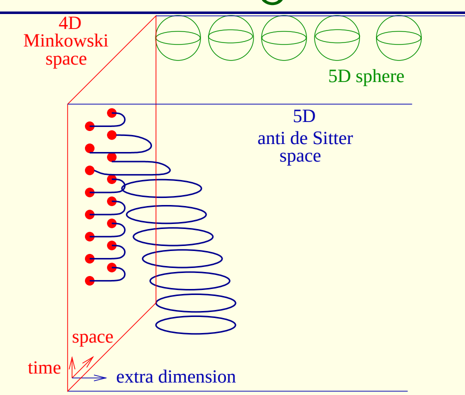
$$\frac{2}{g_{YM}^2}\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^2-\frac{1}{2}(D\Phi)^2+i\overline{\Psi}\not{D}\Psi+V\right]$$

$$V(\Phi,\Psi)=\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi]$$

szimmetriák



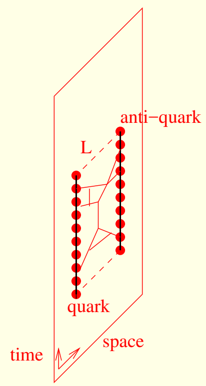
szuperhúr $AdS_5\times S^5$ háttéren



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

AdS/CFT: kvark-anti-kvark potenciál

gluon kicserélődések

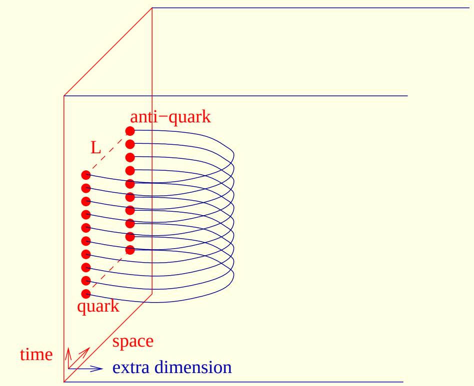


$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$

Feynman gráfokkal 4 hurok

$\equiv$

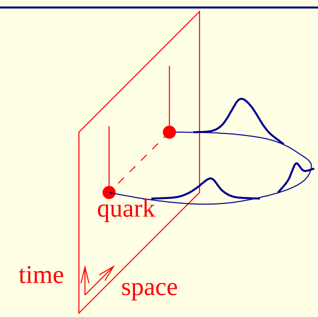
Minimális AdS felület



$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}\left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimális AdS felület+fluktuációk

Egzakt leírás minden csatolásra



gyenge csatolás = mértékelmélet

erős csatolás = húrelmélet

Összegzés

Le vagyunk láncolva a 3 (+1) dimenziós Minkowski világhoz

Az extra dimenziókra csak
az egyenleteink szimmetriáján
keresztül következtethetünk.
Lehet, hogy van több
ekvivalens leírás, de mi
a legegyszerűbbet választjuk.